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ENERGY AND INSTABILITY IN THE ATMOSPHERE

INTRODUCTION

Today as never before Meteorology is faced with innumerable problems, each of which urgently requires a solution. These problems range from the jet stream and momentum transfer to diffusion and turbulent motion; from radiation and stimulated drop growth to forestry meteorology and radio wave propagation. You may then ask why I have chosen to speak tonight on Instability, a subject on which we have long since reached a satisfactory degree of understanding. With the exception of the literature on convection cells and entrainment little has appeared on this subject in the last ten years. I am excepting now that phase of instability known as dynamic instability.

The fact is that instability forms the basis of an important and highly interesting field for investigation which, after rapid development under inspired leadership in the past, has shown little progress in recent years. Tonight I wish to explore with you some of the lesser known aspects of instability in the atmosphere and indicate a line of approach which shows promise of future development. Certainly there is no lack of problems involving the mechanism of instability. One could easily name a score. It appears that the condition of the atmosphere which we call instability is intricately interwoven with all weather phenomena and evaluation of instability must form an integral part of the preparation of any forecast.

Having dwelt on the reasons for this study on the question of instability it might be well to consider briefly what is implied by this term. Qualitatively, I think we all feel that instability implies a tendency for spontaneous development of some condition. Thus the child who attempts to reach the jam pot on the top shelf by balancing himself on a pile of chairs and books experiences a condition of unstable equilibrium. In the atmosphere it may appear as the tendency for a rising parcel of air to rise more rapidly or for a falling parcel to fall more rapidly. It is interesting to note how reasoning such as this has led us to determine criteria for instability in the atmosphere in terms of buoyancy forces whereas in mechanics instability is always defined in terms of energy. This is perhaps excusable in view of the complexity of the energy problem when applied to a compressible fluid on a rotating sphere in contrast with the simplicity of the energy problem in particle mechanics. Nevertheless, the reliance on buoyancy principles has placed restrictions on the thinking of meteorologists which have not yet been overcome in spite of the outstanding contributions of Max Margules and Sir Charles Normand in the field of energy transformation.

THE BUOYANCY APPROACH TO INSTABILITY

The classical approach to instability study has been by way of perturbation considerations and buoyancy forces. This is true of both standard methods of attacking instability problems, the parcel method and the slice method. In the parcel method a small parcel of air is given an infinitesimal displacement in the vertical. As a result of the change in pressure it suffers the parcel experiences a change in temperature in accordance with the adiabatic law. In general this temperature will differ from that of the new environment. Buoyancy considerations then determine whether the parcel will continue to move in the direction of the initial perturbation or return to its original position, i.e., whether the condition is unstable or stable. The same principles apply in the case of the slice method. Here we consider a circulation embodying a rising and a descending current of air. Each current suffers a change in temperature according to the adiabatic law. It is generally assumed that the temperature change satisfies the dry adiabatic law in the descending column and the saturated adiabatic law in the ascending column. From buoyancy considerations one may determine whether the circulation will continue in the same sense or tend to reverse its direction, i.e., whether the condition is unstable or stable.

Both of these methods suffer from rather severe restrictions and neither can be expected to provide a completely satisfactory answer to the problem of instability. This is apparent, for example, from the fact that the parcel method, on the one hand, deals only with a vertical column of air, while the slice method, on the other hand, deals only with the behaviour of the air in a horizontal slice. The methods are also unsatisfactory in that they fail to consider the energy budget of the atmosphere. One may argue that the energy which is available for parcel method motion can be computed from areas on a thermodynamic diagram. This, however, is begging the issue since, in those considerations, energy is introduced in the form of the work done and appears as an afterthought and not as a fundamental concept.

THE ENERGY APPROACH TO INSTABILITY

Let us now observe this problem of instability from another vantage point, using energy considerations as a basis for our criterion. Consider a parcel of air of unit mass in the atmosphere. What is the total energy contained in this parcel of air? Some of its energy will be potential, due to the work done against gravity in raising the parcel to its present level. Some of it will be kinetic, due to its absolute motion in space. A great amount of it has been provided by the heat needed to raise the temperature from an arbitrary initial zero state to the existing state. Some of this heat has gone to increase the internal energy, $C_v\ (T-T_o)$, where C_v is the specific heat at constant volume and $(T-T_o)$ is the change in temperature. Some has been used to do work in expansion against the pressure forces, $p\alpha$, where p is the pressure and α is the specific volume. The sum of these represents the total amount of heat energy available. This is represented by h, the specific enthalpy. Thus $h = C_v\, T + p\alpha +$ const. Or one may consider the heat to have been added at constant pressure so that $h = C_p\, T +$ const.

In moist air account must be taken of the ability of water to change phase from solid to liquid to vapour. For a gram of ice associated with a parcel of air at the arbitrary zero of temperature and the appropriate pressure to become a gram of water vapour at temperature T and partial pressure e, one can imagine a process similar to that described above for dry air. Heat is applied to the ice at fixed external pressure to raise its temperature and change its phase and volume until it attains the desired final gaseous state. Because much of this heat is absorbed as latent heat of fusion and evaporation, the formula for the exact expression of the heat given to the water vapour is not so simple as for dry air, but one recognizes that the amount of heat energy absorbed is the total heat, or enthalpy, of the unit mass of water vapour in its final state, which is as definite an attribute of the vapour as its temperature or partial pressure.

The importance of enthalpy as a concept lies in the fact that it embodies all of the energy of a parcel of air, apart from the kinetic and the potential. Now potential energy is of little importance as an energy source in a fluid such as the atmosphere since whenever a parcel of air rises another must sink to take its place. It appears then that the specific enthalpy is the thermodynamic potential for the atmospheric engine. This means that, neglecting energy which may be received from outside the system, any increase in kinetic energy must come as a result of a decrease in enthalpy.

One of the first investigators to realize the importance of the enthalpy concept in meteorology was Margules (1906). While he never mentioned it by name Margules made great use of this idea. His approach was by way of a system in which the air was held in a closed container. Typical examples of Margules are shown in Figs. 1(a) and 2(a). Figure 1(a) represents a situation in which we have an airmass of uniform entropy S_2 superposed on an airmass of uniform entropy S_1. It is assumed that S_1 exceeds S_2 so that an unstable situation exists. Figure 2(a) represents a similar situation in which the entropy changes continuously from S_1 to S_2. The most stable (thermodynamically) states are represented by Figures 1(d) and 2(d). Margules gives an analysis in which the systems pass through the intermediate states shown. Some of these states have special meteorological interest in themselves. Figure 1(c), for example, represents a condition similar to an ideal frontal discontinuity.

As the air in Margules bottles readjusts itself according to the method described the enthalpy of the system is lowered and this drop in enthalpy is manifest as an increase in kinetic energy. One could profitably discuss the configurations into which the wind so developed must be organized but time does not permit a consideration of this aspect of the energy transformation process here. This brief description of Margules approach is of interest here for two reasons. Firstly because it is, perhaps, the earliest intensive discussion of the problem of instability in the atmosphere. Secondly because the concept of enthalpy was first applied to meteorological problems in this work.

Fortunately, the laborious computations of Margules are no longer necessary for determination of released energy. The introduction of thermodynamic energy diagrams has made possible the graphical solution of such problems and, through the inspired work of Normand, has led to a better understanding of the physical processes involved. I should like to turn now to the later work of Normand in the problem of energy conversion and instability in the atmosphere.

Let us consider the question of how enthalpy drop appears on the tephigram. In Figure 3 is shown an extended tephigram in which the isotherm O_1M represents an arbitrarily selected temperature for which the enthalpy shall be considered to be zero. To find the specific enthalpy corresponding to some state, say that of the point A, we must determine the heat input which would be required to raise a parcel of unit mass from the state represented by the point O_1, isobarically to the existing state, i.e., the integral of Tds for the path O_1A, where T and s represent temperature and entropy, respectively. On the tephigram this integral has a value equal to the area O_1AM. Thus for considerations involving dry air the enthalpy changes for specified processes may be computed from a simple geometrical consideration of the changes in the area corresponding to those processes. Furthermore we are not restricted to processes involving the motion of single parcels of air but may extend the computations to the most elaborate motions of air parcels. Thus all of Margules examples are susceptible to this attack, designated by Normand as the multiple parcel method. By a slight adjustment in the reasoning the effects of moisture in the atmosphere may also be appraised. In a series of papers Normand (1937a) (1937b) (1938a) (1938b) (1946) has provided the solutions to a great variety of complex problems. These discoveries of Normand rank, I think, amongst the most fundamentally significant in the history of modern meteorology and a review of some of his examples by way of illustration will be given here.

Consider first a column of dry air in which all parcels except the one at the base have the same entropy. We will suppose the parcel at the base to have a somewhat higher entropy than the others. This, indeed, is a special case of the problem of Margules illustrated in Figure 1(a) where the lower airmass is reduced to the status of a small parcel. On the tephigram shown in Figure 3 this example corresponds to the line CD plus the single point A. We know, of course, that the situation is highly unstable but let us apply the enthalpy reasoning to determine the quantity of enthalpy available for the production of kinetic energy. To do this let us suppose that the parcel at A rises adiabatically to the top of the column, i.e., to the point B. During this process the system loses a quantity of enthalpy equal to $O_1AM - O_2BM$ = area O_1ABO_2. Now the space vacated by the parcel at A does not remain a vacuum. Rather the remaining parcels of the column must subside precisely the right amount to give a continuous column of air. Thus the parcel represented by E drops to the state D and the enthalpy of the system increases by a corresponding amount. Similarly other parcels such as those represented by F and C must drop. The integrated increase in enthalpy of the system due to the motion of all these parcels must be equal to the area O_1DCO_2. The net loss of enthalpy due to the motion of all the parcels will be equal to the area ABCD. This well-known result is usually obtained by other reasoning. The approach described here serves to show how the enthalpy concept may assist us in considering problems of instability. It has the further virtue of showing that, contrary to popular opinion, the environment does move during the parcel method process. In fact the movement of the environment must be considered in the computation of the energy change if we are to derive the correct result.

A somewhat more normal situation in which the entropy decreases continuously with height is illustrated in Figure 4. This corresponds to Margules example shown in Fig. 2(a). Applying similar reasoning to this case we find that the energy released as the lowermost parcel rises to the top of the column is equal to the area shaded in the diagram. It is apparent that the motion of this single parcel and the subsequent drop in the environment does not exhaust the available energy of the system. A more complete discussion of this problem will follow later.

At this point we might profitably consider the representation on the tephigram of changes in enthalpy during motion in which a change of state of the water content of the air takes place. The important thing to be noted in this connection is that a change in state, such as condensation or evaporation, which takes place under conditions of constant pressure will result in a change of temperature of the parcel, because of the latent heat released or absorbed, but will not alter the enthalpy content since no heat has been added to, or subtracted from, the system as a whole. Consider the change in enthalpy of a saturated parcel of air, represented by a point A on the tephigram shown in Fig. 5, as it passes through the changes of state indicated by the saturated adiabat AL. One may consider the motion to proceed by a succession of small discrete jumps. Thus the parcel might rise from A to B without condensation. Since the specific heats for dry air and moist air differ only slightly the change in enthalpy will be the same as for dry air and may be computed in the manner already established. Of course, the parcel will now be supersaturated. If the upward motion is suspended momentarily, and condensation is permitted to begin and continue until the air is once more saturated, the temperature will rise due to the release of latent heat and the parcel will assume the state indicated by point C on the diagram. This change will not, however, be accompanied by a rise in the enthalpy of the system. The changes of enthalpy as the parcel moves by successive jumps from state A to state L, must equal the shaded area on the diagram. This fact permits us to extend the enthalpy reasoning to unstable systems involving moist air, and to introduce other concepts such as latent instability. Application of the reasoning to the situation illustrated on the tephigram in Fig. 6 shows that if a parcel of air having dry and wet bulb temperatures corresponding to A and D, respectively, rises through the atmosphere to the level of B a quantity of energy equal to the shaded area between A and B must be supplied. On rising further to the level of C energy equal to the shaded area between B and C will be made available for conversion to kinetic energy.

The preceding examples were intended to illustrate the method by which the enthalpy concept may be applied in the study of some conventional situations. We will now demonstrate the versatility of the method in handling more complicated situations in which a very considerable readjustment of the air parcels takes place. While the method applies equally well to moist air we will content ourselves with a study of examples involving dry air. The cases selected for study correspond to certain of Margules examples and the analysis is due to Normand.

Consider first the case of a superadiabatic lapse rate in dry air. Margules would have considered it as air in a container, as in Fig. 2(a), with continuously decreasing entropy from S_1 at the foot to S_2 at the top. As the air overturns in the container the parcels slide past each other until the final configuration as illustrated in Fig. 2(d) is reached. On the tephigram the original situation is represented by the line AC (Fig. 7). The final situation, achieved after overturning, is represented by the line DB. Taking two units of air at A and C to exchange position the drop in enthalpy is equal to the area ABCD. Taking successive pairs of units at regular intervals of pressure, successively smaller areas are obtained. For example, the interchange of parcels at P and R, where the pressure interval from A to P equals the pressure interval from R to C, yields the area PQRS. Considering a third dimension of mass in our diagram we may pile these areas one on the other to obtain a pyramid of height M/2 where M is the total mass of the system. The factor 2 in the denominator derives from the fact that pairs of parcels were used in each interchange. The energy drop is given by the volume of the pyramid, illustrated in the lower diagram of Fig. 7. Strictly speaking the sides of this pyramid will not be plane surfaces in view of the fact that the pressure lines on the tephigram are not equally spaced. This deviation from linearity will have little effect on the volume, however, since the concavity on the low pressure side is approximately balanced by the convexity on the high pressure side. The energy drop is therefore $(M/2)(1/3)$(area ABCD) or $(1/6)$(ABCD) per unit mass. It should be emphasized once again that the descending parcels play as big a role in the energy change process as the rising parcels since one is inclined to concentrate on the rising air and neglect the contribution of the other.

Another interesting application of the multiple parcel method is afforded by the problem illustrated in Figure 1(a), in which two airmasses, each of uniform entropy are superposed, the upper mass having the lower entropy. We have already discussed the particular case where the lower airmass is reduced to a single parcel of unit mass. We now consider the case where the airmasses are of equal mass. The situation is illustrated on the tephigram in Fig. 8, where the line AB represents the lower mass and CD the upper. A variety of geometrical configurations may be obtained depending on the nature of the process visualized. Since all of these must give identical numerical results we will discuss only the simplest approach. First, we permit an exchange of the unit masses at A and C. The energy so released is equal to the area AECF. Continuing in the same manner we reach points P and R. When these exchange positions an amount of energy equal to the area PQRS is released. As in the preceding example the total enthalpy drop is given by a pyramid of height M/2. The shape of this pyramid is shown in the lower diagram of Fig. 8. It follows that the energy released as the airmasses exchange position is equal to $(M/2)(\frac{1}{2})$(area AECF) or $(\frac{1}{4})$(AECF) per unit mass.

A further example of some special interest is the one represented in Fig. 1(b). Initially we have two airmasses of different entropy, but of uniform entropy within themselves, lying side by side. The final state of minimum enthalpy is illustrated in Fig. 1(d) where the mass of larger entropy lies above the mass of lower entropy. It is apparent that the initial state for this problem is the half-way state for the problem just discussed so that the energy released must be one half of that computed for that problem. However, as an exercise in the use of the multiple parcel method it is worthwhile considering the question on its own merits. On the tephigram the initial state is represented by the two lines AB and DC, Fig. 9. The final state is the curve DFEB. One may begin with an interchange of the parcels at A and C, following with successive pairs till the parcels at E and F are reached. The energy released is given by a pyramid, similar in shape to that found in the last problem but having a height of only M/4 since only half of the parcels have been involved in the process. The enthalpy drop is therefore equal to (1/8)(ABCD) per unit mass.

As a final example of the generality of the multiple parcel method it will be shown that the so-called slice method of instability analysis may be derived as a special case of the former. Fig. 10 shows the tephigram representation of the problem. In the slice method we take a thin horizontal slice through a circulation system in which air is rising through an area A_1 at a speed v_1 and, in other portions, is sinking through an area A_2 at a speed v_2. A basic assumption of the slice method is that conditions should be baratropic initially. To satisfy this condition we assume two identical columns of air side by side. For purposes of representation these have been separated slightly and appear as the lines HK and H'K' in Fig. 10. It is further assumed that upward motion follows the saturated adiabatic law while downward motion follows the dry adiabatic law. The curve containing points DBGF illustrates the state of the parcels taking part in the circulation a short time later, when the upward moving air has moved through a distance $\triangle z$. Thus the parcels originally at E, C, and A, have moved to F, D, and B, respectively. In this diagram it has been assumed that the ascending air is moving twice as fast as the air which is descending. To generalize the matter the number of downward steps (n) in the layer of thickness $\triangle z$ must be given by $n = v_1/v_2 = A_2/A_1$. It is evident that the energy change, for the two mass units, due to the motion of these parcels within the slice, is the same as would have occurred had E moved to F and A moved to D, i.e., AEF - AED. Since we are interested in a criterion for instability we may express the favourable condition as AEF - AED > 0 for instability. From the geometry of the figure it follows that, to a high degree of approximation AF > ED = AG.

$\therefore T_F > T_G$ for unstable equilibrium. It is, however, possible to obtain a relationship between these temperatures in terms of the lapse rates and the constants of the process.

$$T_F = T_E - \gamma_s \triangle z$$
$$T_E = T_C + \gamma \triangle z / n$$
$$T_C = T_D - \gamma_d \triangle z / n$$
$$T_D = T_G + \gamma \triangle z$$

where $\gamma, \gamma_d, \gamma_s$ are, respectively, the lapse rate, the dry adiabatic lapse rate, and the saturated adiabatic lapse rate. From this it follows that the criterion for instability is

$$T_F - T_G = (\gamma - \gamma_s)\triangle z - (\gamma_d - \gamma)\triangle z / n > 0$$

which reduces to the familiar slice method criterion

$$\gamma > \frac{A_1 \gamma_d + A_2 \gamma_s}{A_1 + A_2}$$

While this derivation is more complex than the usual slice method approach it does serve to show that the slice method is a rather special case of the more general multiple parcel method which is applicable to any closed system.

FLUID ENERGY APPROACH TO INSTABILITY

The methods discussed so far are strictly thermodynamic. In applying them one must always consider the air to be at rest or undergoing a very restricted form of motion. Account is not taken of the fact that, in general, under true atmospheric conditions, the air is already in motion. Even in the multiple parcel method the parcels are pushed around paths selected by the analyst and the air is not permitted full hydrodynamic freedom.

In this section I will describe an approach to the problem which combines hydrodynamic and thermodynamic principles in a more natural fashion. This theory may be considered as an extension of the work of Margules and Normand although it has not, as yet, been nearly so thoroughly exploited. For purposes of this illustration we must make use of the third equation of motion in the form of an energy equation.

$$\frac{d}{dt}\left(\frac{1}{2} w^2\right) + wg + \frac{w}{\rho}\frac{\partial p}{\partial z} = 0$$

The three terms in this equation represent respectively the rates of change of kinetic energy, potential energy, and enthalpy. That the last **term** represents the rate of change of enthalpy due to motion in the upward (z) direction can be readily shown. The equation for specific enthalpy is

$$h = u' + p\alpha + \text{Const.}$$

where u' is the specific internal energy. The first law of thermodynamics may be written

$$dq = du' + p\,d\alpha$$

where dq is the quantity of heat added to unit mass of the system. Both of these equations hold for all systems regardless of composition and hence for moist air. It follows that

$$\frac{dh}{dt} = \frac{dq}{dt} + \frac{1}{\rho}\frac{dp}{dt}$$

where $\rho = 1/\alpha$ is the density.

For adiabatic processes

$$\frac{dh}{dt} = \frac{1}{\rho}\frac{dp}{dt}$$

Now $\partial p/\partial t \equiv 0$ in this equation since any energy change due to motion of the pressure field results from outside influence and not from motion of the parcel. Using this fact we find that the rate of change of enthalpy due to motion in the vertical is

$$\left.\frac{dh}{dt}\right|_z = \frac{w}{\rho}\frac{\partial p}{\partial z}$$

Therefore the third term in the energy equation:

$$\frac{d}{dt}\left(\frac{1}{2}w^2\right) + gw + \frac{w}{\rho}\frac{\partial p}{\partial z} = 0$$

represents the rate of change in enthalpy. This equation can now be used to determine a criterion for instability. To do this we note that for motion to take place under conditions of unstable equilibrium it is necessary that kinetic energy be created at a continually increasing rate. For instability in the vertical our condition for unstable equilibrium becomes:

$$\frac{d^2}{dt^2}\left(\frac{1}{2}w^2\right) > 0$$

Introducing the energy equation quoted above we find

$$-\frac{d}{dt}\left(\frac{1}{\rho}\frac{\partial p}{\partial z} + g\right) - w\frac{d}{dt}\left(\frac{1}{\rho}\frac{\partial p}{\partial z}\right) > 0$$

Applying the energy equation and the equation of state we obtain

$$\left(\frac{dw}{dt}\right)^2 - w\frac{d}{dt}\left(RT\frac{\partial \ln p}{\partial z}\right) > 0$$

$$\therefore \left(\frac{dw}{dt}\right)^2 - wR\frac{\partial \ln p}{\partial z}\frac{dT}{dp}\frac{dp}{dt} - wRT\frac{d}{dt}\left(\frac{\partial \ln p}{\partial z}\right) > 0$$

$$\therefore \left(\frac{dw}{dt}\right)^2 + wR\frac{\partial \ln p}{\partial z}\frac{RT}{g}\frac{dT}{dz}\frac{d\ln p}{dt} - wRT\frac{d}{dt}\left(\frac{\partial \ln p}{\partial z}\right) > 0$$

Noting as before that $\partial \ln p/\partial t = 0$ the condition becomes

$$\left(\frac{dw}{dt}\right)^2 - wR\frac{\partial \ln p}{\partial z}\frac{RT}{g}\gamma_c\left(u\frac{\partial \ln p}{\partial x} + v\frac{\partial \ln p}{\partial y} + w\frac{\partial \ln p}{\partial z}\right) - wRT\left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)\frac{\partial \ln p}{\partial z} > 0$$

where $\gamma_c \equiv -dT/dz$, the dry or saturated adiabatic lapse rate. Introducing the hydrostatic equation this reduces to

$$\left(\frac{dw}{dt}\right)^2 + wR\gamma_c(w - u\tan\alpha - v\tan\beta)\frac{\partial \ln p}{\partial z} - \frac{wg}{T}\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) + \frac{w^2 g\gamma}{T} > 0$$

where $\gamma \equiv -\partial T/\partial z$, the temperature lapse rate

$\tan\alpha \equiv$ the component of slope of the isobaric surface in the x- direction,

$\tan\beta \equiv$ the component of slope of the isobaric surface in the y - direction.

$$\boxed{\frac{w^2 g\gamma}{T} > 0}$$

$$\therefore \left(\frac{dw}{dt}\right)^2 - \frac{wg\gamma_c}{T}(w - u\tan\alpha - v\tan\beta) - \frac{wg}{T}\left[u\left(\frac{\partial T}{\partial x}\right)_p + v\left(\frac{\partial T}{\partial y}\right)_p - u\frac{\partial T}{\partial z}\tan\alpha - v\frac{\partial T}{\partial z}\tan\beta\right] + \uparrow$$

This reduces to

$$\left(\frac{dw}{dt}\right)^2 - \frac{wg}{T}\left[u\left(\frac{\partial T}{\partial x}\right)_p + v\left(\frac{\partial T}{\partial y}\right)_p\right] + \frac{wg}{T}(\gamma - \gamma_c)(w - u\tan\alpha - v\tan\beta) > 0$$

Assuming quasi-geostrophic conditions so that $u \doteq -g/f\tan\beta;\ v \doteq g/f\tan\alpha$

the criterion for instability takes the simple form

$$\left(\frac{dw}{dt}\right)^2 - \frac{wg}{T}\left[u\left(\frac{\partial T}{\partial x}\right)_p + v\left(\frac{\partial T}{\partial y}\right)_p\right] + \frac{w^2 g}{T}(\gamma - \gamma_c) > 0$$

This discussion serves to show that when the atmosphere is permitted full hydrodynamic freedom of motion the criterion picks up dynamic terms, including solenoid terms, and thus varies from the classical condition $\gamma > \gamma_c$.

CONCLUSION

To sum up I would say that through a discussion of the earlier work on Instability I have tried to show the value of the energy concepts introduced into this field by Margules and Normand. These principles are of wider application than the present problem but indicate, I think, that progress can be made only through energy considerations. In addition I have tried to point the way for further development of the Instability problem through the introduction of hydrodynamic considerations. While some results have been obtained much remains to be done before we have a reasonably complete picture of the processes involved.

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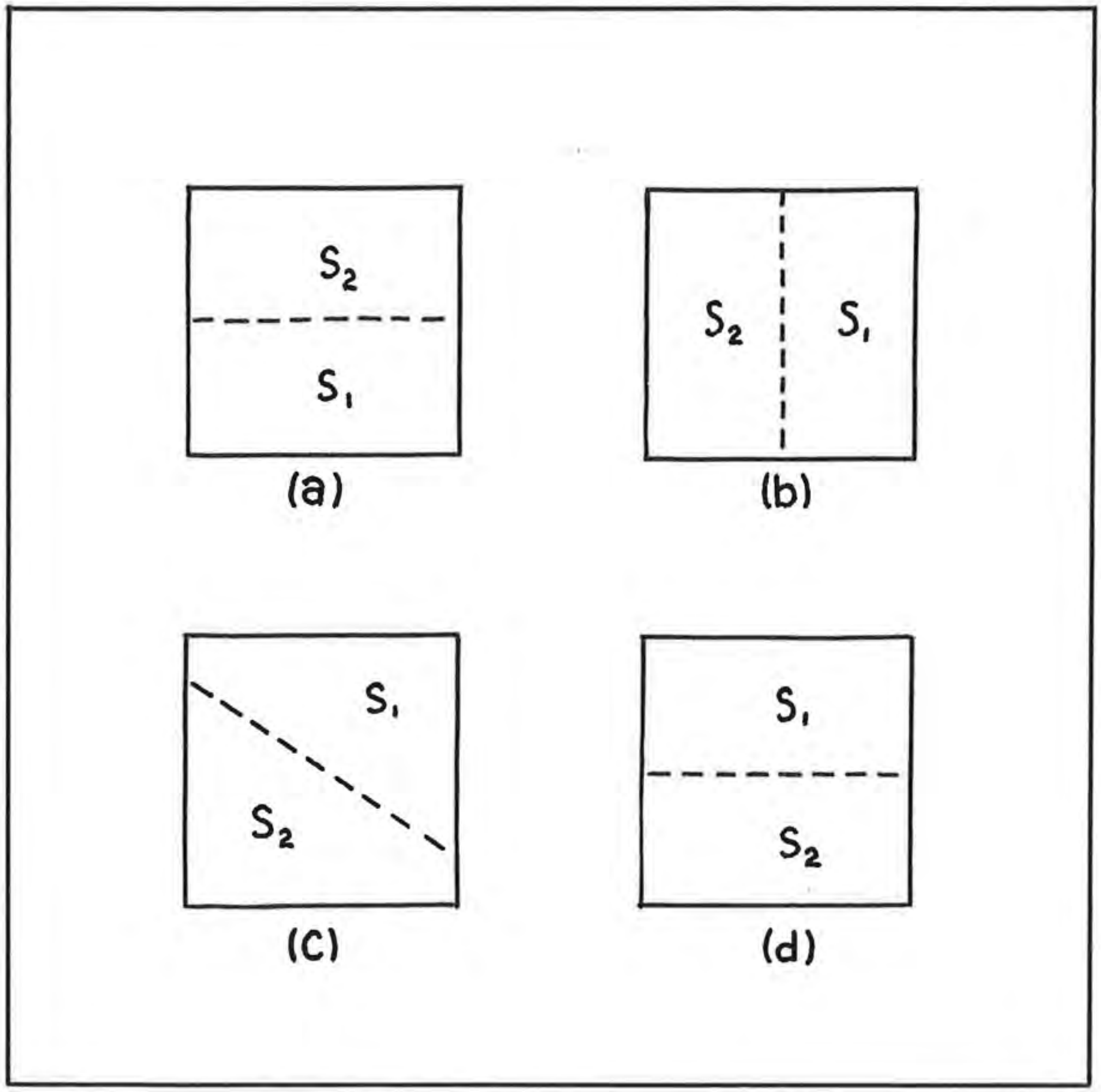

Figure 1

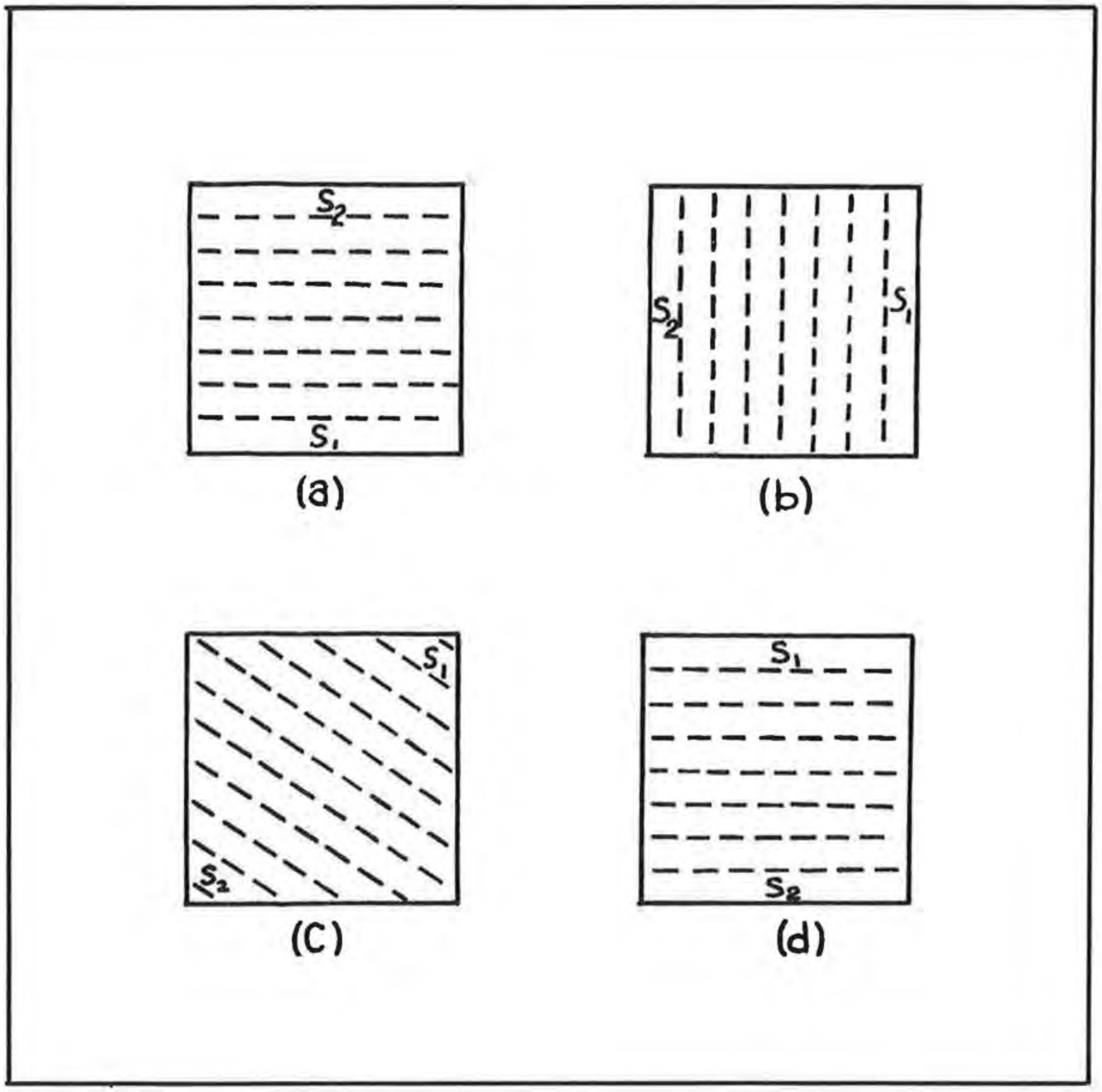

Figure 2

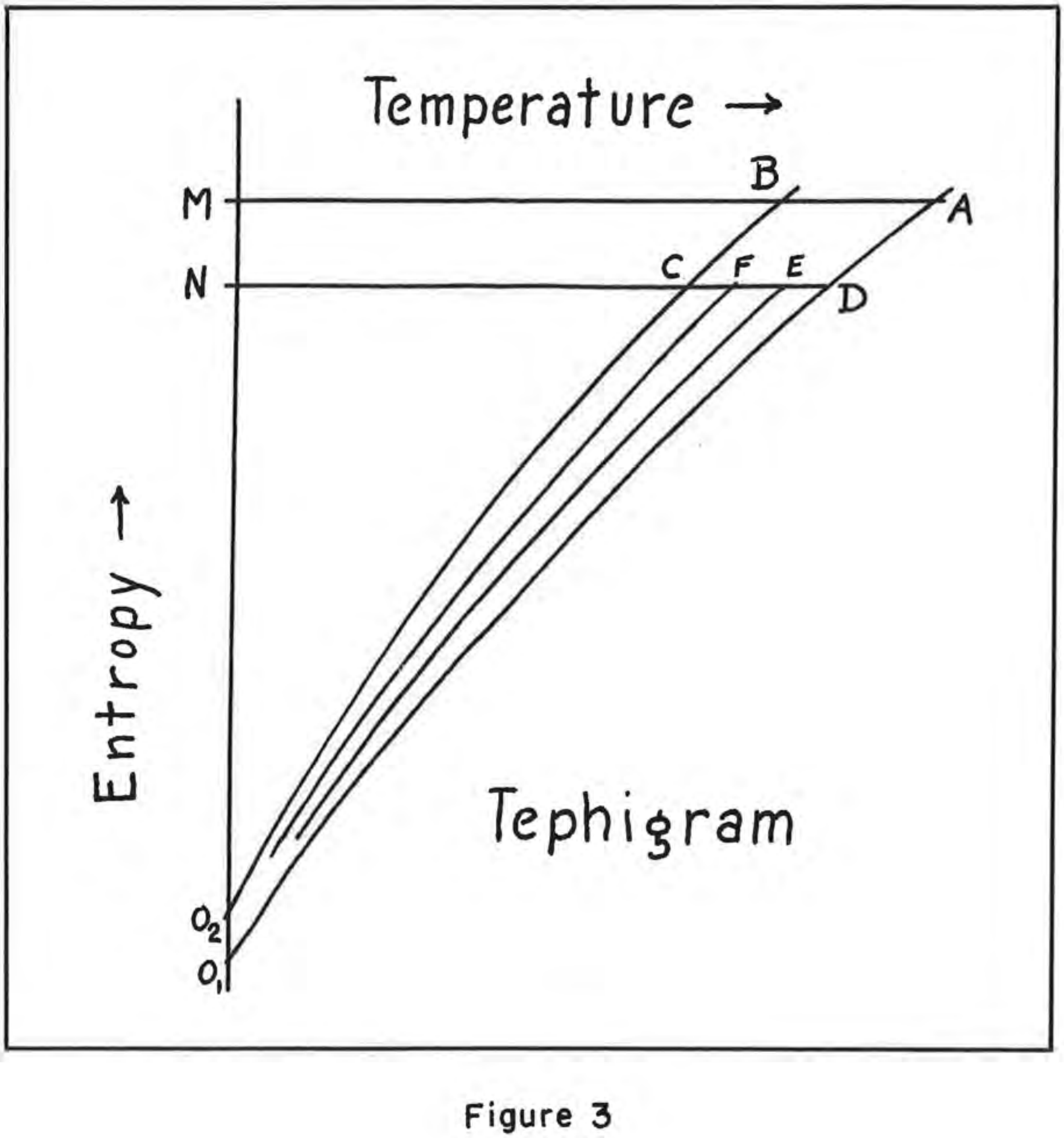

Figure 3

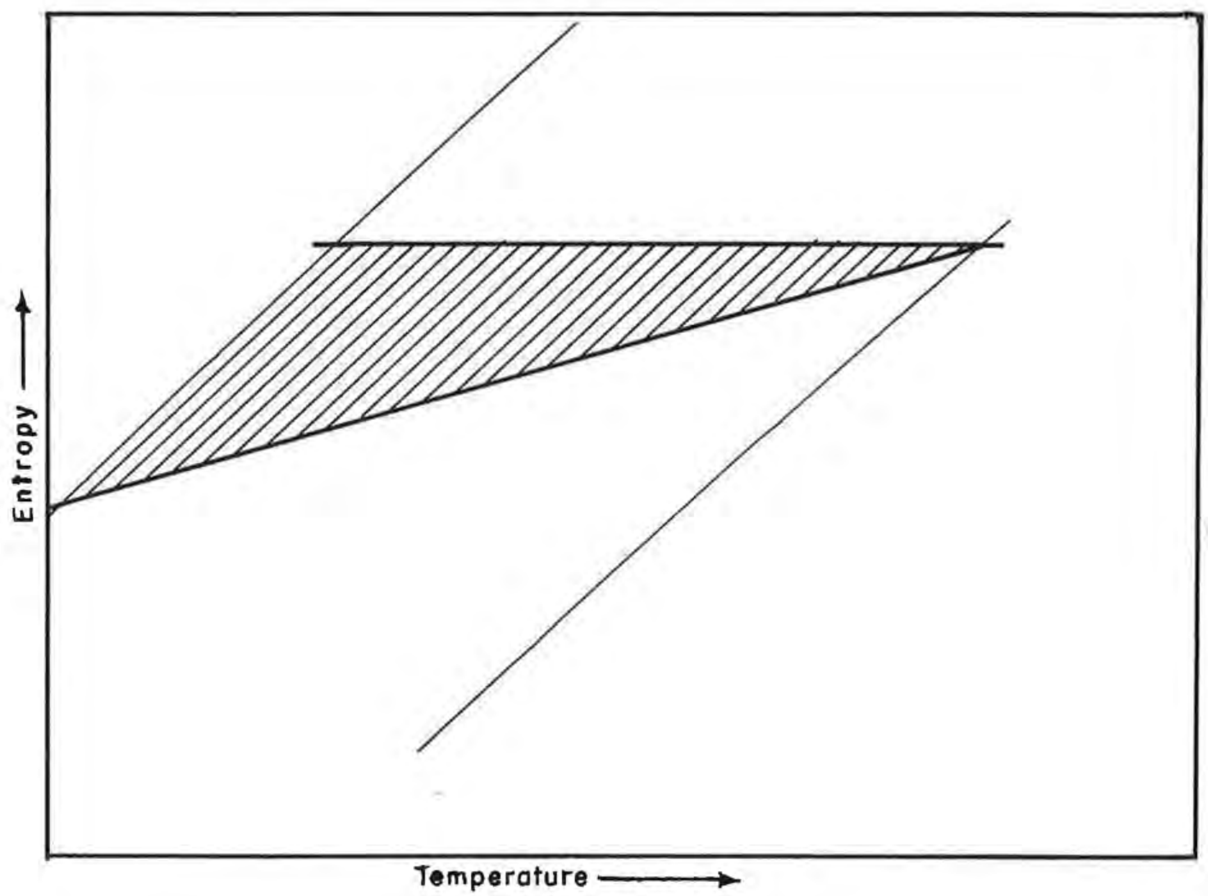

Figure 4

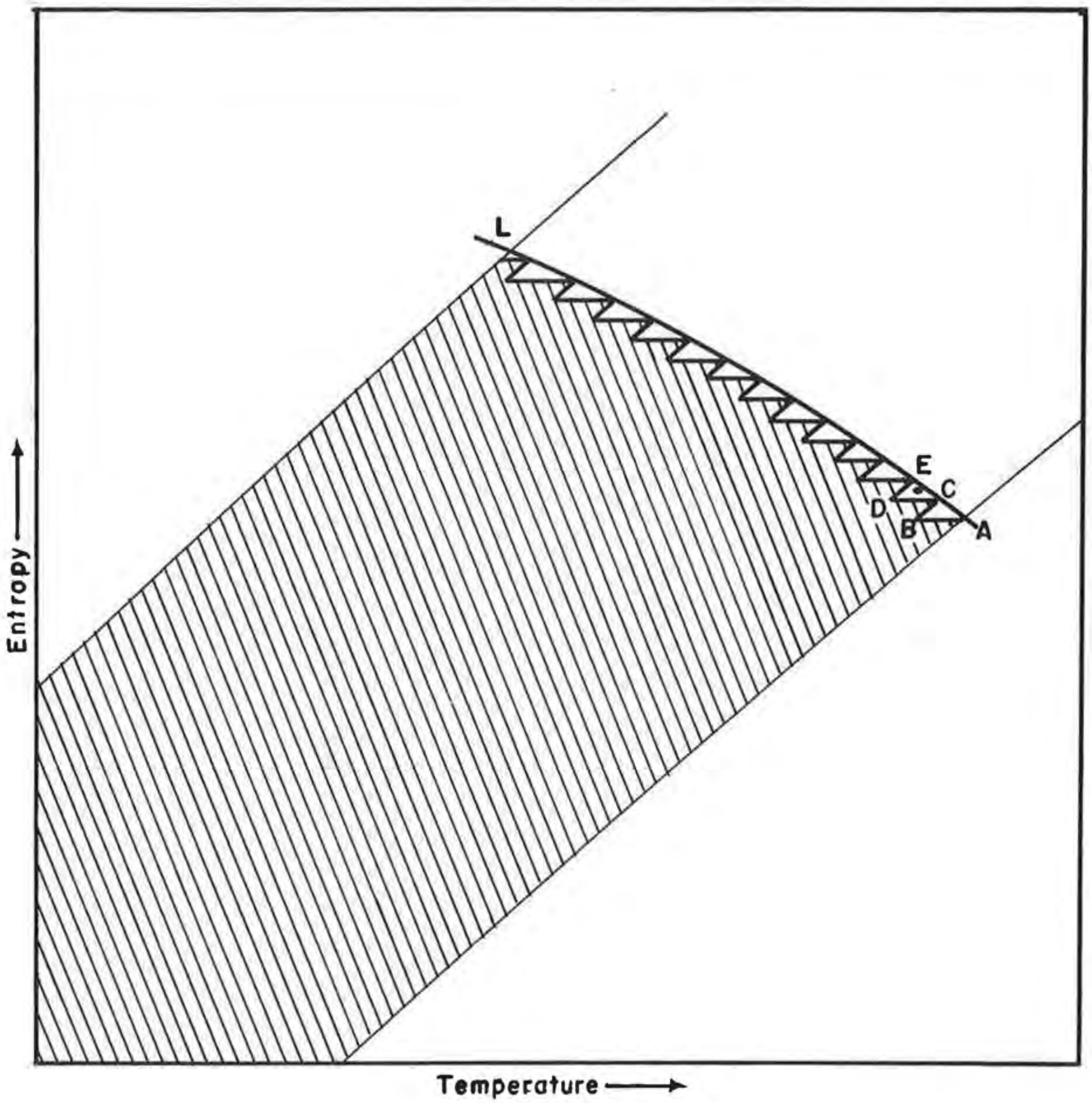

Figure 5

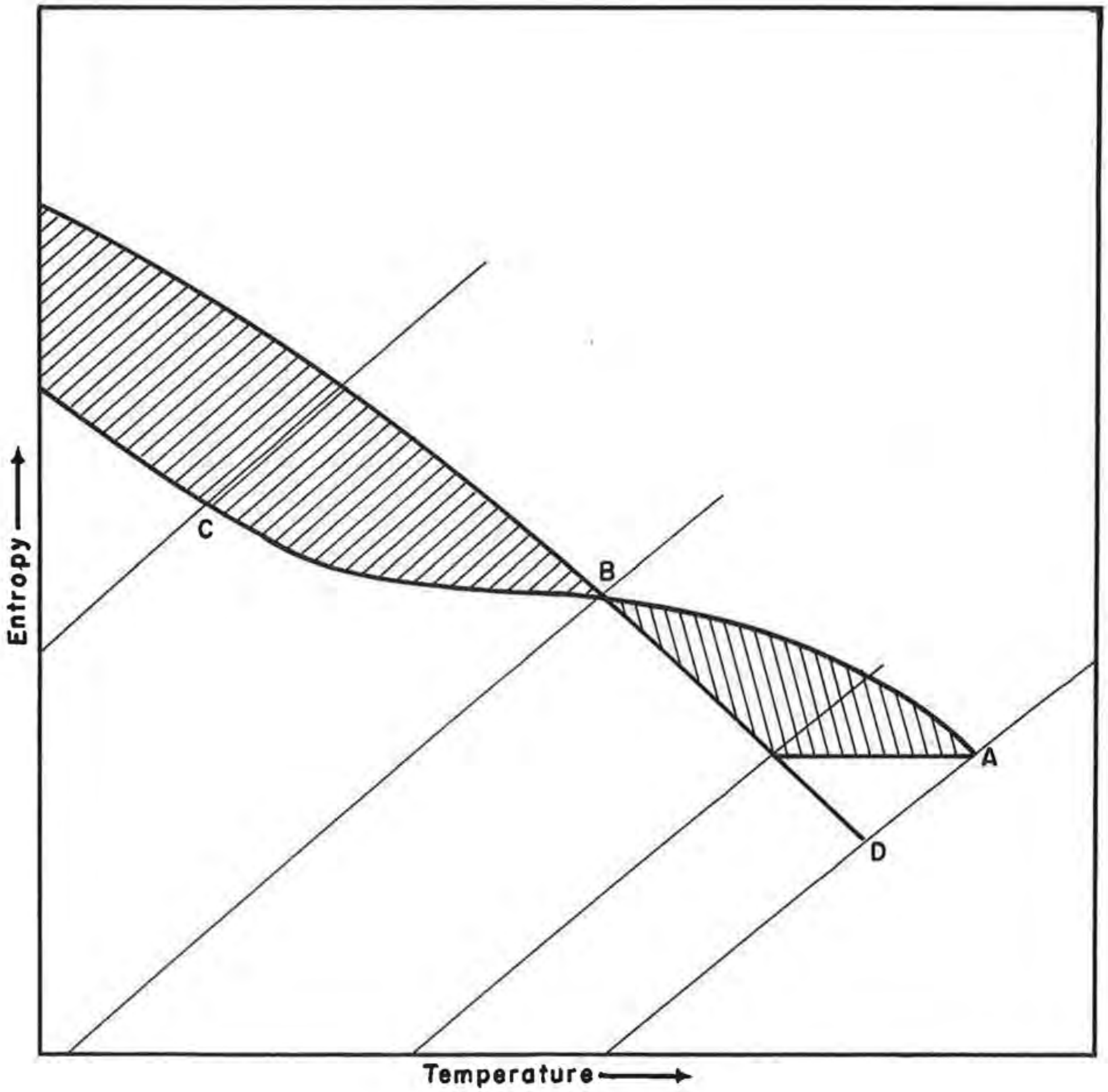

Figure 6

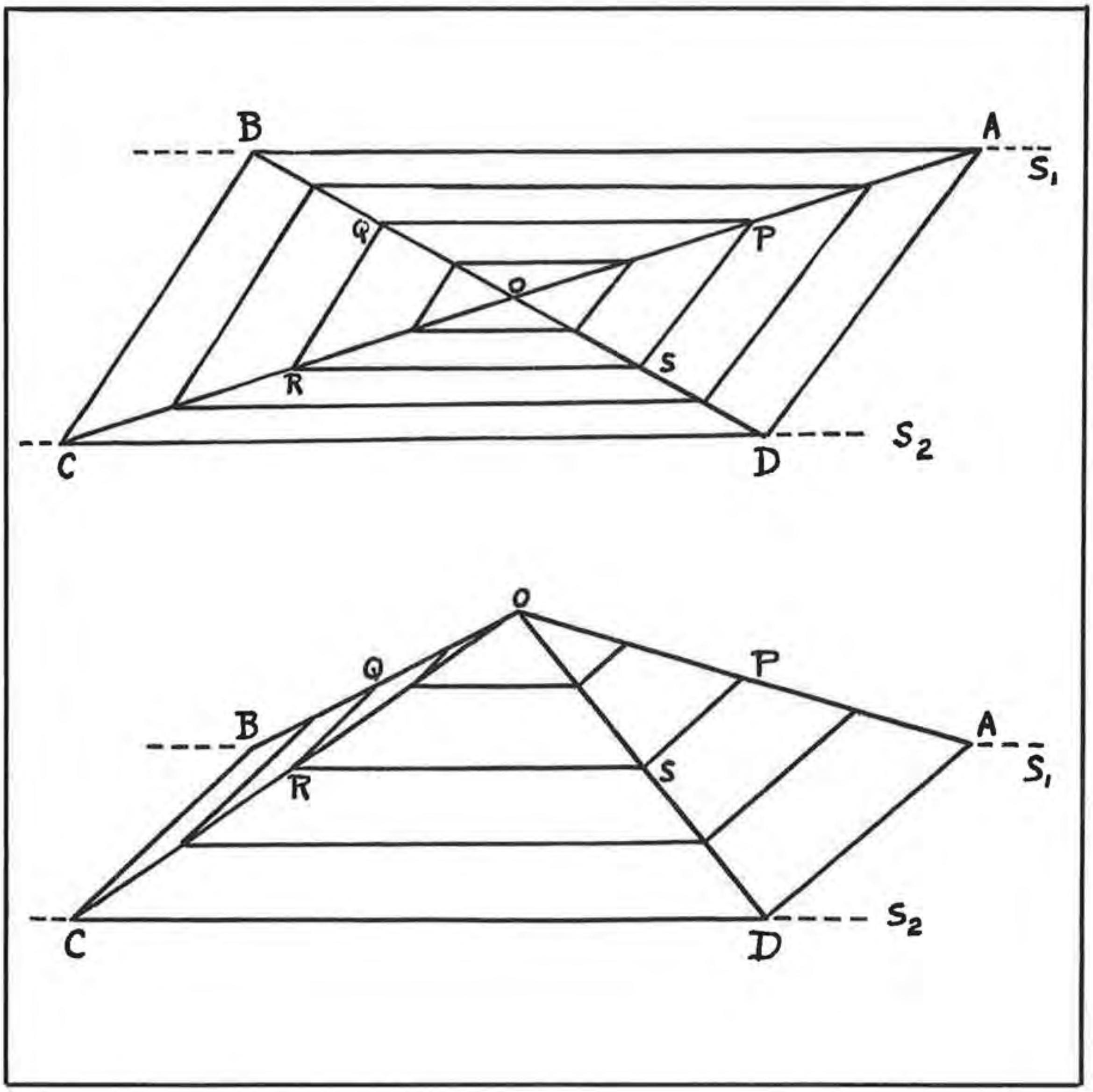

Figure 7

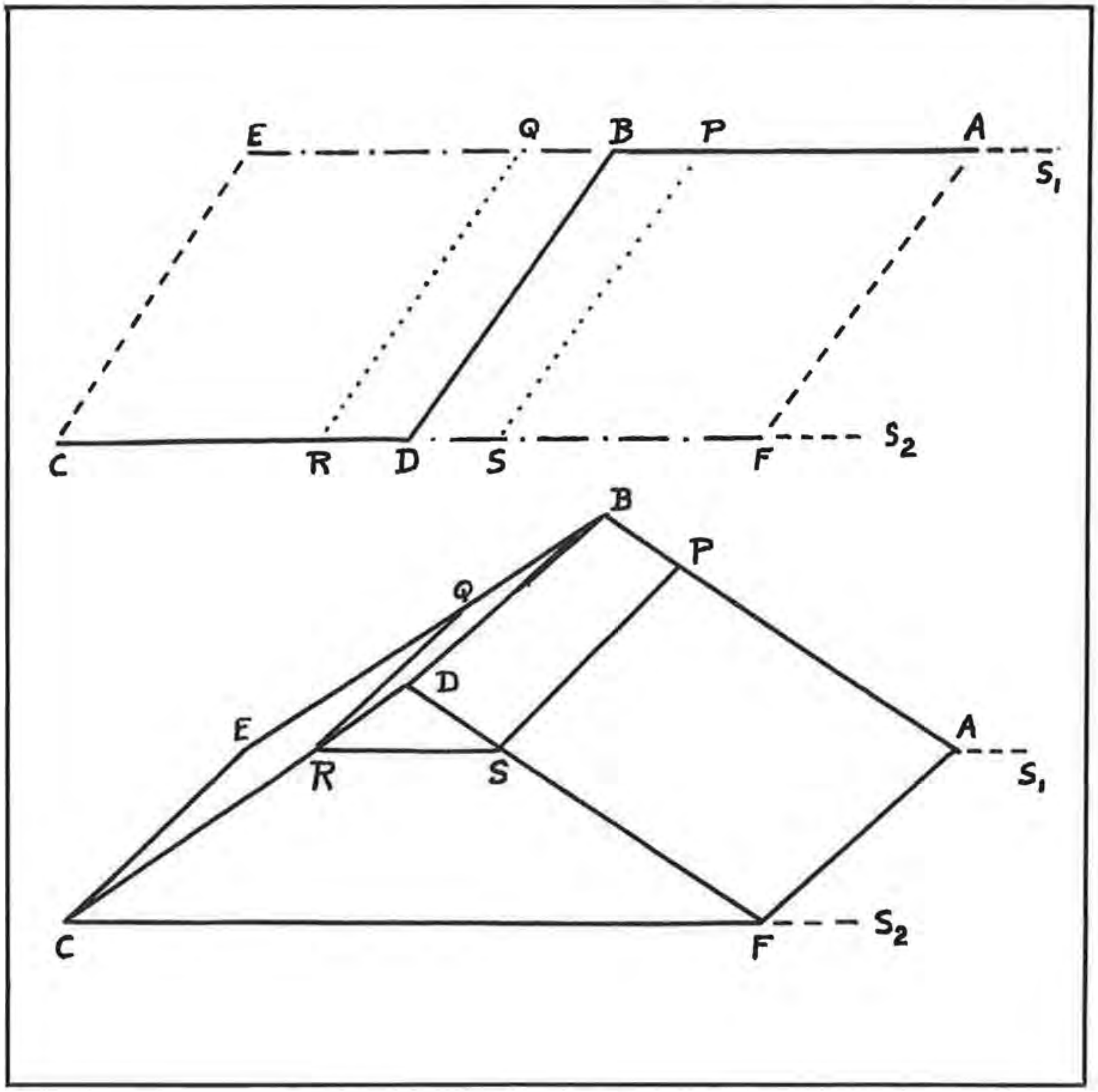

Figure 8

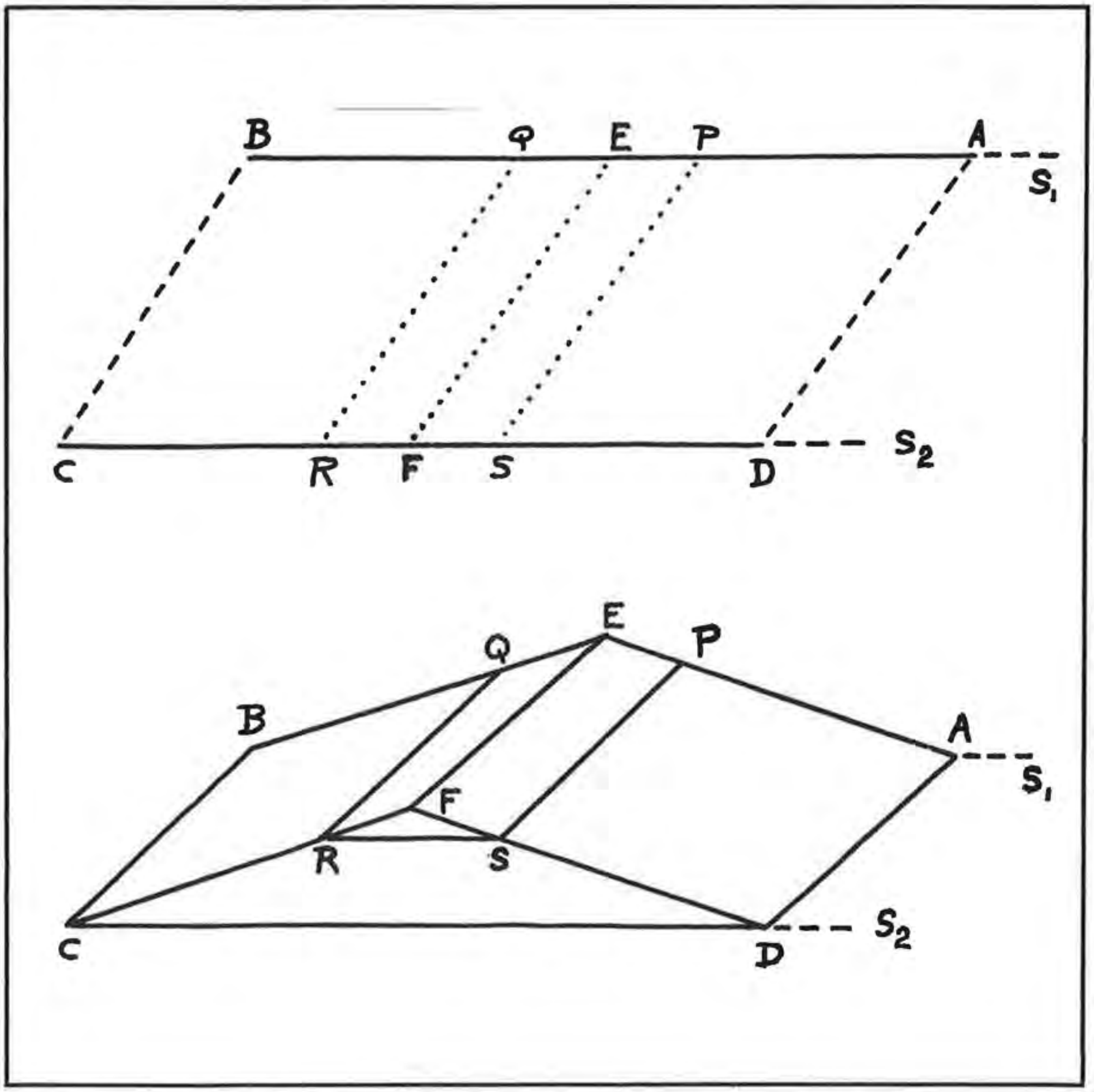

Figure 9

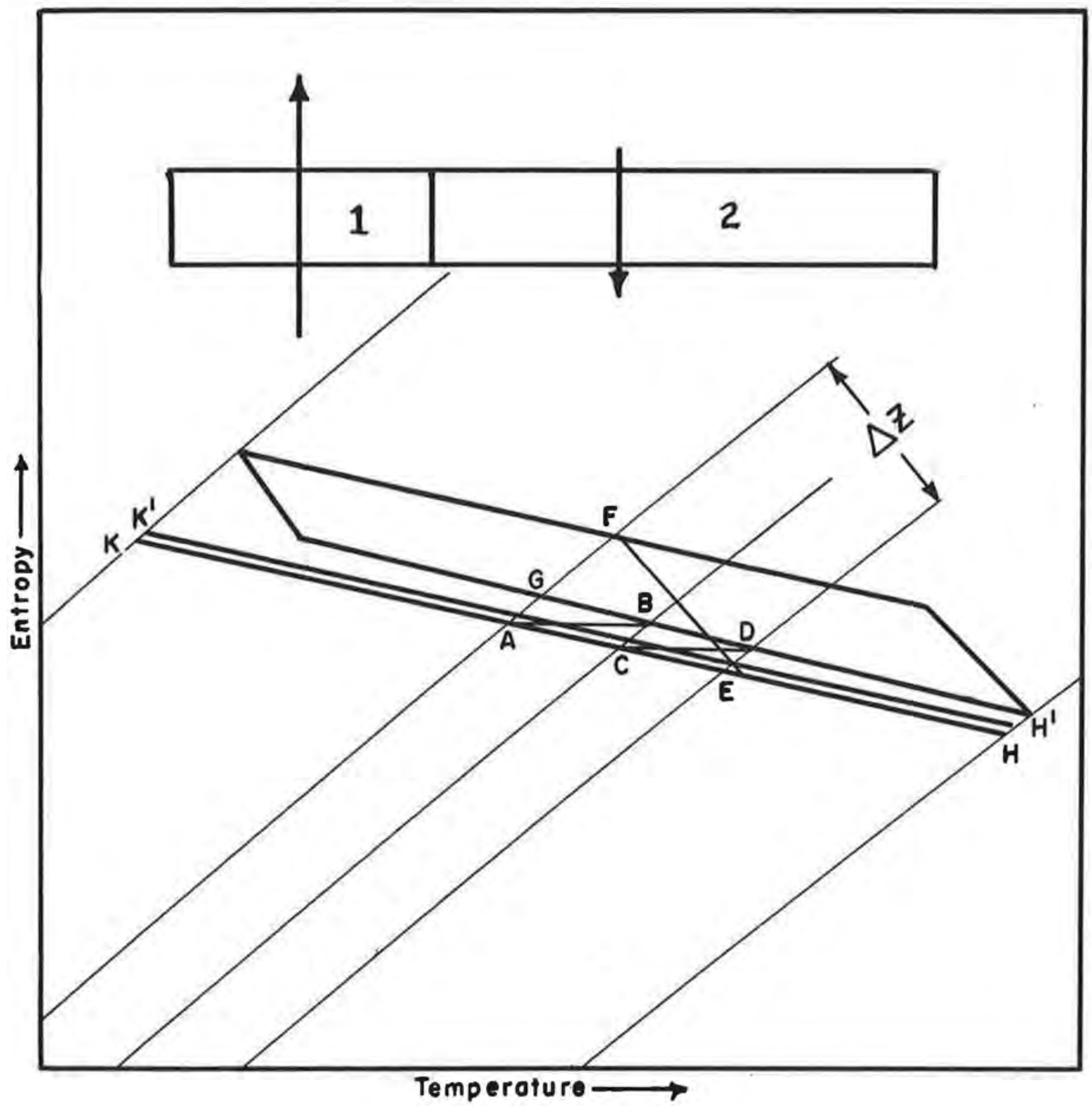

Figure 10