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CLIMATOLOGICAL BULLETIN

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AN ESTIMATION TECHNIQUE FOR SNOW SURFACE ALBEDO

by

D. E. Petzold*

Introduction

In recent years, it has become quite evident that alternative energy sources will be required to meet our future energy needs. The realization has been made that no other source is as unlimited and possesses as great a potential as solar energy. One obvious problem facing the implementation of a solar energy system during winter is the relative abundance of cold and minimally sunny days in Southern Canada (Sasaki, 1975). Perhaps first thought of as a deterrent to a successful implementation of a solar heating system, the often present winter-time snow cover has been shown to substantially augment the solar radiation received on a south-facing vertical wall (or collector) by forward scattering from the snowpack (Hague and Werren, 1976). The proportion reflected forward is of course dependent upon the physical characteristics of the snow surface. The magnitude of the effect of these surface properties on solar radiation is determined by its shortwave reflectivity or albedo, the unitless ratio of reflected to incident solar radiation on a surface.

Snow surface albedo (α) cannot be neglected in the estimation of net radiation over a snowpack (Petzold, 1974) and thus it is a parameter necessary for the successful calculation of the surface heat exchange and melt rates of a snowpack (Price, 1977; Taylor-Alt, 1975). Petzold has also shown that a snow surface can exhibit a high day to day variability in α which is in sharp contrast to that of a naturally vegetated surface. Since

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snow can frequently be expected to cover land surfaces for six months northward of 45°N, an exact knowledge of snow surface albedo will be essential for the successful implementation of snow melt predictions, energy budget determinations, or perhaps solar collector utilization. Unfortunately, there is still a dearth of permanent radiation stations that measure the components necessary for the computation of albedo across Canada (Petzold, 1974).

Widely used albedo estimators developed by the U.S. Army Corps of Engineers (U.S.A.C.E.)(Gray, 1970) assume a new snowfall to have an albedo of no greater than 0.84, even during the accumulation season. Their method relies solely on the age of the snow surface. Climatological research in both the north and south polar regions has produced sound relationships between albedo and cloud amount, solar elevation, and snow surface density, thus the age of snow.

It is the purpose of this paper to collect and consolidate these relationships and determine the feasibility of incorporating them into one albedo prediction model capable of producing a greater accuracy than the U.S.A.C.E. estimator.

The Effect of Cloud Cover on Albedo

High albedo values under conditions of diffuse radiation from overcast skies have been observed repeatedly in both north and south polar regions (Liljequist, 1956; Barge, 1971; and Holmgren, 1971). Albedo generally increases with increasing cloud cover and opacity and decreasing cloud height. Thus, the albedo of a surface under a dense stratus overcast is expected to be considerably higher than that of the same surface under a fibrous cirrus overcast, which has a characteristically low opacity.

Relations between albedo and cloud cover for four polar stations are shown in Figure 1. In each case, the relation was derived from radiation data of an entire summer field season. Data from these stations include measurements over all snow surfaces expected in a more temperate zone during winter, from a fresh powder cover to a melting snowpack in spring. Visual inspection of Figure 1 indicates a similar variation at all four stations.

Obviously, since albedo will vary with every different snow surface type, it is not possible or practical to estimate an absolute value

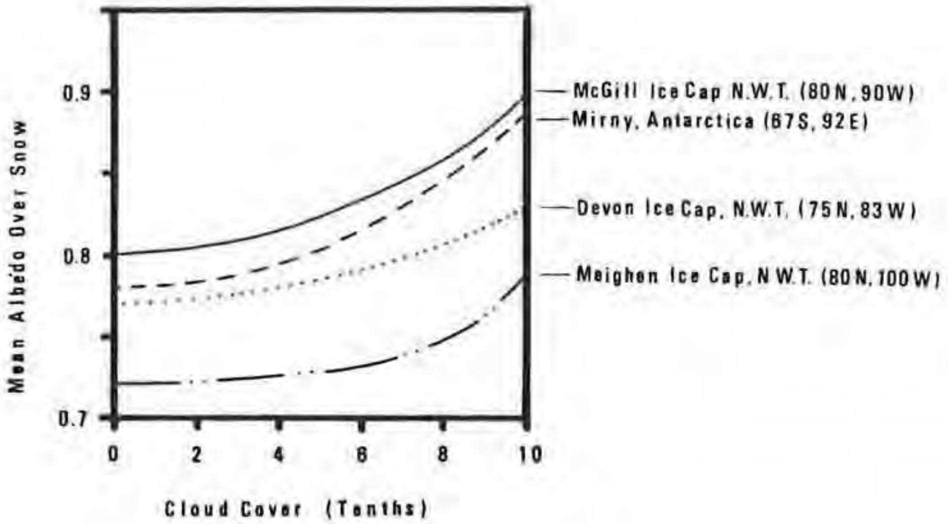


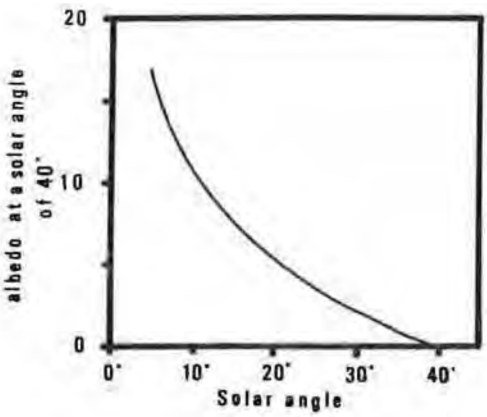
Fig. 1. Mean albedo variations with cloud cover at four polar stations (derived in part from Havens, 1964; Rusin, 1961; and Holmgren, 1971).

of albedo by knowing the cloud cover for an arbitrary snow surface. However, from these four stations, an average percent change in albedo can be related to cloud cover amounts. Regression analysis using the data plotted in Figure 1 yields the relation:

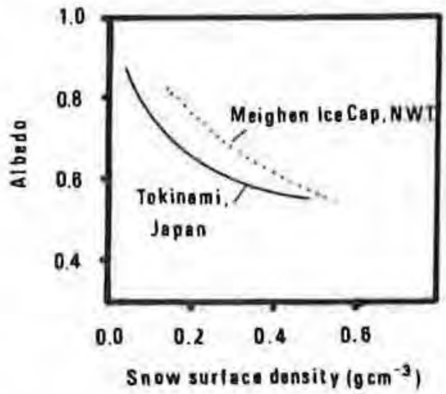
$$\left(\begin{matrix}\text{\% change from} \\ \text{clear sky albedo}\end{matrix}\right) = 0.449 + 0.0097\ (\text{Cloud amount})^3, \qquad (1)$$

for cloud amounts expressed in tenths, from one to ten. This relation has a correlation coefficient of 0.999 and a standard error of prediction of 0.2 percent. Thus, if the albedo is known for day 1, knowledge of the cloud cover on day 2 will permit albedo estimation for that day. Take for example, day 1 with a mean albedo of 0.86 under clear skies. Given the same surface conditions for the following day under a cloud cover of 6/10, the albedo for this day would be increased by 0.026 above the clear sky albedo, according to equation (1). Thus on day 2, a mean α of approximately 0.89 would be expected, due solely to the mean cloudiness for that day.

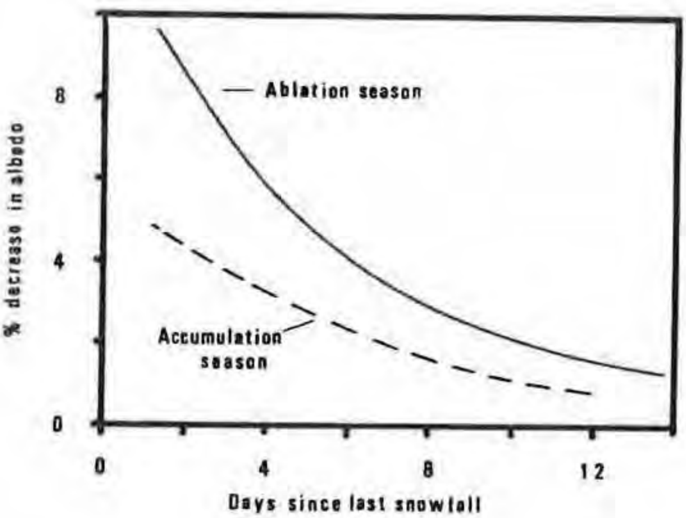
Conditions responsible for the explanation of this phenomenon are as follows. Investigations by Liljequist (1956) indicate that albedo values for the infrared part of the spectrum are markedly lower than for visible light. With a dense overcast, solar radiation, predominantly



(a) Function of Solar Angle



(b) Change with Snow Surface Density



(c) Relative to Previous Day as a Function of Time since Last Snowfall

Fig. 2. Percentage changes in albedo.

diffuse, will be relatively rich in visible light due to multiple reflections between the highly reflective snow surfaces and cloud bases. The fact that snow has a high albedo for visible light is one of the reasons for the observed increases in albedo as cloud amount, and particularly opacity, increases.

With clear skies, solar radiation is predominantly direct rather than diffuse. Under these conditions, surface roughnesses cast shadows. Separate surface snow grains are reached by a direct beam only on one side. On the other hand, overcast skies create no shadows and diffuse radiation reaches surface grains on all sides and from all directions. For this reason, the albedo under clear skies can be expected to be lower than that for overcast conditions.

Albedo Variation with Solar Angle

Under clear skies or scattered and high level clouds, the albedo of a snowpack decreases with an increase of the solar angle to about 40°. Under dense low level clouds and for solar elevations greater than 40°, solar angle has little if any influence on albedo (Liljequist, 1956; Rusin, 1964).

The most obvious use of a solar angle - albedo relationship would be for the investigation of diurnal albedo variations. However, the use of mean daily albedoes obviates constant use of this relation. But, slight as it may be, the constant increase of the sun's elevation from winter through summer solstice in the Northern Hemisphere is great enough to affect the albedo of snow where snow is on the ground for a major part of that period.

Published observations from Antarctica by Rusin and Liljequist will be the basis for obtaining a quantitative dependence of albedo on solar altitude. According to measurements summarized by Rusin (1964), albedo decreased by 8 to 20 percent under clear skies as the solar angle increased from 10° to 45°. This result was obtained for several antarctic stations.

Again, a relationship was derived in such a way that a percent change in albedo is expressed as a function of a solar (noon) angle. Because it is generally agreed that there is no such dependence for solar angles greater than 40°, the percent change of albedo relating to a certain solar angle is expressed as a value relative to an albedo at an elevation of 40°. Thus, according to the relation shown in Figure 2a, the albedo of a clear day with a maximum solar angle of 15° is 7.4 percent higher than that of a clear day when the solar angle is 40° or greater. For instance, consider a snowpack at 45° latitude; the maximum solar angle varies from 22° on December 22 to 45° on March 20. Thus, during this period, a decrease in albedo by approximately 0.05 would be expected, due solely to this factor.

Unfortunately, a simple mathematical expression for the relation shown in the figure has not been found.

Snow Surface Decay

If a snowpack is left undisturbed and its surface is not altered by a new snowfall, a natural decay of the surface will take place that will affect its albedo. Generally, a gentle compaction occurs after a snowfall, due to the destruction of fine snow flakes and columns. As well, there will be an accumulation of minute dust particles from the atmosphere and fine, wind-blown debris from nearby vegetation. At the surface of a melting snowpack, in addition to settling and debris accumulation, a physical change from powder to granular snow takes place due to surface melting. Thus the albedo of a melting snowpack will change much more rapidly than that of an accumulating pack. A well-correlated relationship exists between albedo and surface density of snow (Arai, 1966). This relation, shown graphically in Figure 2b, was developed from observations taken in the ablation season of the Tokinami River basin, Japan (36°N, 144"W). A definite trend is suggested by the fact that albedo-density data collected during three field seasons on Meighen Ice Cap, N.W.T. (80°N, 100°W) agree with the Arai relationship.

The U.S. Army Corps of Engineers (Gray, 1970) expresses this natural albedo deterioration as a simple exponential decay function, one for a snowpack in the accumulation season and one for a ripe snowpack in the melt season. The information presented on the graphs of these decay functions was used to obtain a relationship between the daily percent change in albedo and age of the snow since the last snowfall. For an accumulating snowpack (maximum air temperature < 0°C),

$$\left(\begin{matrix}\text{Daily \% change}\\ \text{in albedo}\end{matrix}\right) = -10^{(0.78 - 0.069\ D)} \qquad (2)$$

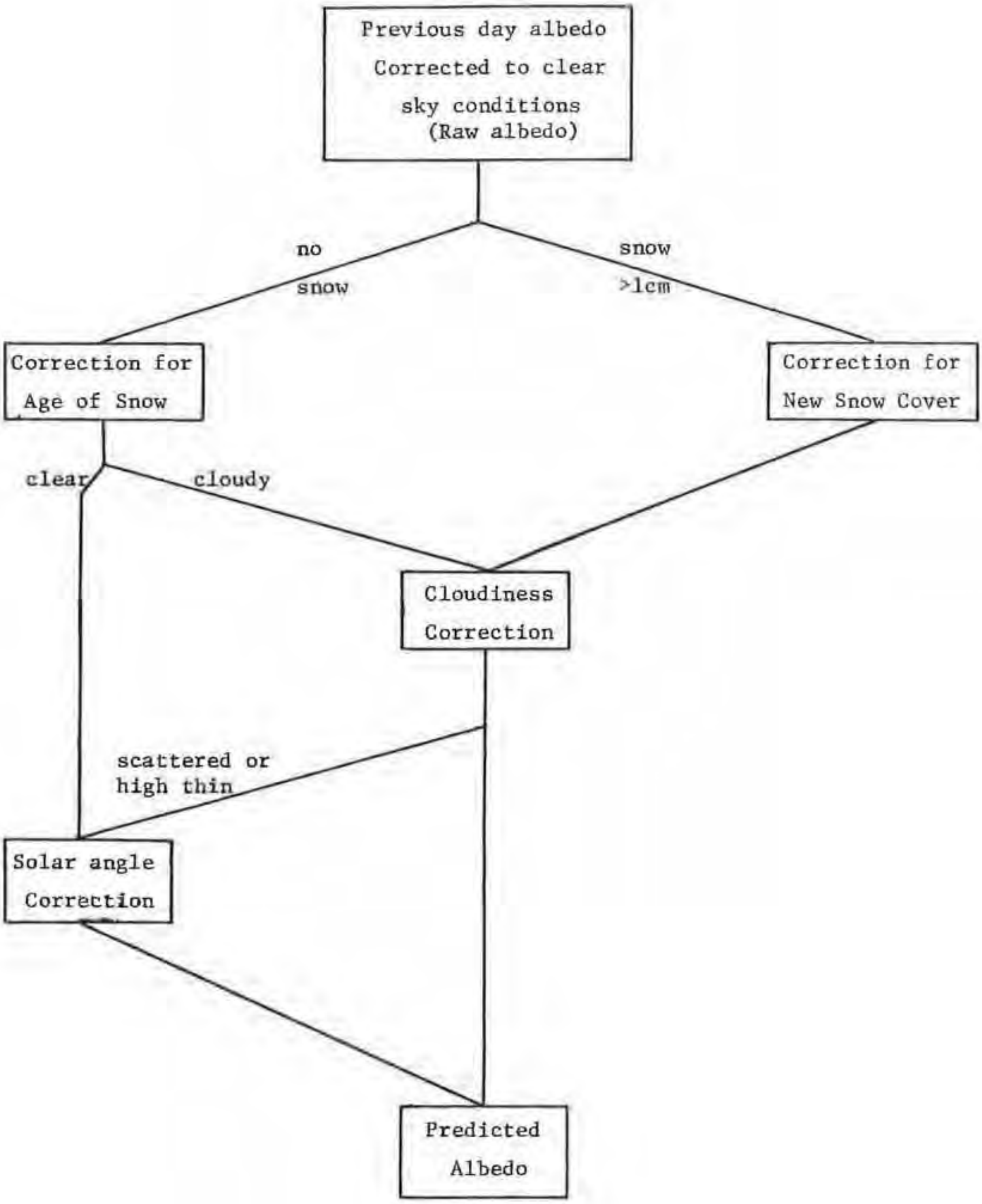
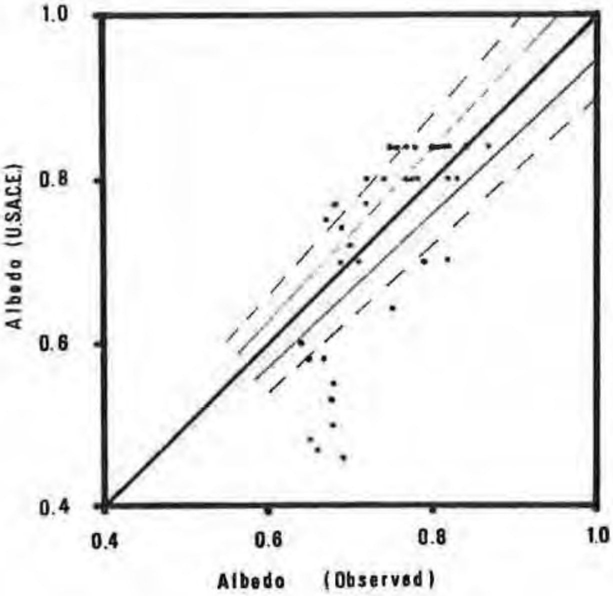
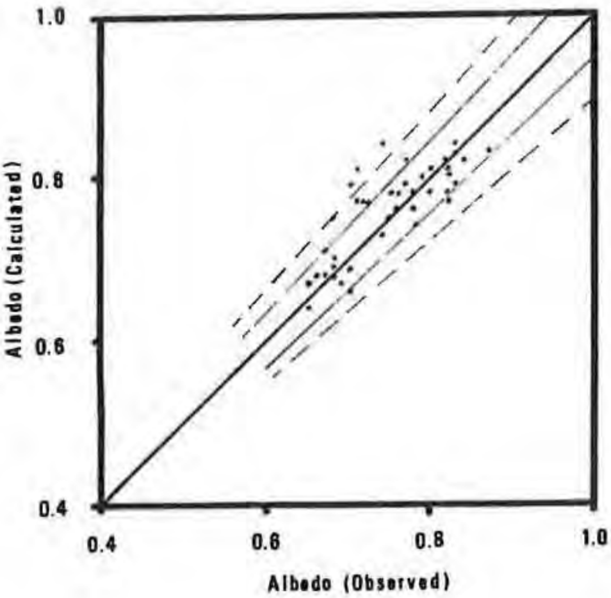


Fig. 3. Albedo Reconstruction Flow Chart



(a) U.S.A.C.E. estimations



(b) Calculated albedo.

Fig. 4. Observed and Predicted albedo.

and for a melting snowpack (maximum air temperature > 0°C),

$$\left(\begin{array}{c}\text{Daily \% change}\\ \text{in albedo}\end{array}\right) = -10^{(1.05 - 0.070\ D)} \qquad (3)$$

where D is the number of days since the last snowfall. These relations are presented graphically in Figure 2c. It must be kept in mind for clarity that the percent change in albedo calculated in equations (2) and (3) is relative to the previous day's albedo.

A new snowfall will of course reverse the effect within the duration of the precipitation. It has been my personal experience while collecting climatological data on Meighen Ice Cap that a new snow cover of at least one centimeter is necessary to completely obliterate the influence of the underlying old snow surface.

Results of the Computer Prediction

The previously discussed relationships were incorporated into one model for albedo prediction by computer. Figure 3 indicates the possible steps in an albedo reconstruction for one day.

Input into the prediction model includes the mean daily temperature, mean cloud cover (tenths), and predominant type (low, middle or high), the sun's elevation at solar noon, and the number of days since the last snowfall greater than one centimeter (0 indicates new snowfall). For successful operation of the model, certain steps must be followed. First is that an actual mean daily albedo must initially be taken over the surface in question on the day of a new snowfall. Lacking this, it is quite reasonable to assume that a new snowfall exhibits an α value of between 0.84 and 0.89, due to the uniformity of new snow. For low sun angles, this value is likely to vary between 0.90 and 0.95.

According to the relations previously derived, this initial albedo must be "corrected" as if it were taken on a clear day with a solar angle greater than 40°. Then the reconstruction is completed by adding or subtracting the results of the various age, solar angle or cloudiness factors to this raw albedo. The raw albedo depends on the conditions of the previous day and is "reduced" to clear sky conditions. Thus, an error in the first day prediction would mushroom as the age of the snowpack increases. The U.S.A.C.E. decay functions utilized in the present model were found to consistently under-predict the actual albedo, as shown in Figure 4a. To correct for this, a relationship was found between this error and the age of the snow. It was developed for an 11-day period in which no new snow occurred so that the snow surface decayed at a constant rate. The predicted albedo is increased by the following percent:

$$\text{Correction } (\%) = 3.86 + 0.380\, D + 0.123\, D^2;$$
$$r = 0.95, \tag{4}$$

where D is the age of the snow > 0.

This definite relationship between the error and the age of the snow indicates the exponents in equations (2) and (3) are too large, thus accounting for the consistently low predictions produced by the U.S.A.C.E. relations.

The most convenient data available to test the accuracy of this model were obtained from Dr. B. Taylor-Alt for the 1969 summer field season on Meighen Ice Cap, N.W.T. Snow surface albedos were reconstructed for the period July 17 through August 28, 1969. Dr. Taylor-Alt (personal communication) estimates the observed albedos are accurate to within ±5%, due to instrumentation and methods of data reduction. On sunny days, shadows cast by the solarimeters and the supporting structure represent an additional error of -1.4% as determined by Goodall (1971).

The results of the prediction agree very well with the observed values. Of the 43 predictions made, 30 were within ±5% of the observed, only three were greater than or equal to ±10% and three had no observed data for comparison. The greatest errors occurred on days with fresh snow (0, 1 or 2 days old). It is possible that peculiar local conditions, such as rain mixed with snow, caused the lower than expected observed values.

Figures 4a and 4b show a comparison of the albedo predictions from the U.S.A.C.E. graphs and the model presented in this paper, respectively. The U.S.A.C.E. predictions are good for new snow but as the age increases over a melting snowpack (where accurate predictions are perhaps most important) estimations are consistantly low resulting in very high errors. In general, the model developed here reproduces the actual mean daily albedo more consistently than the U.S.A.C.E. method.

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THE EFFECT OF RAINFALL CHARACTERISTICS AND POST-WETTING SYNOPTIC CONDITIONS ON EVAPORATION RATES FROM A WETTED HARDWOOD CANOPY

by

B. Singh*

Introduction

Rainfall intercepted by vegetation and its subsequent loss through evaporation constitutes an integral part of the water balance of forested watersheds. The need to understand its nature and magnitude is therefore understandable. Interceptional loss is the amount of precipitation that is prevented from reaching the ground by the aerial parts of vegetation. This loss is accounted for by two processes: evaporation of water from the canopy during the period of rainfall, and evaporation of intercepted rainfall following the period of wetting. Interceptional loss therefore depends on weather conditions during wetting since they regulate the amount of evaporative loss and on canopy characteristics which determine the retention capacity of the stand. Windiness that produces mechanical shaking of the branches, and the impact velocity of raindrops during severe storms can also lessen the retention capacity of the canopy.

Although the woody parts of vegetation such as branches and stems retain some moisture, by far the greater part of intercepted rainfall is withheld by leaves. Generally a leaf absorbs little, if any, water from the surface (Rutter, 1963). Its storage capacity may therefore be considered to be the amount of water it can retain on its surface. This amount is a function of leaf size, its configuration and composition, together with the viscosity of the water and the external pressure on the

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liquid, as well as the amount of precipitation (Leonard, 1967). The alignment of branches, canopy density and smoothness of the woody parts, such as the bark, are other determining plant factors.

Weather factors, or more precisely rainfall characteristics, also affect interceptional loss in that evaporation of intercepted rainfall can occur during the period of wetting, provided that a vapour pressure gradient exists between the wetted canopy and the ambient air. Wilm and Neiderhof (1941) observed that about 19% of each storm is lost to the ambient air by evaporation from the canopy during rainfall. Rutter et al (1971/72, 1975) found similar values.

In this paper an attempt will be made to show that the amount of concurrent evaporation during rainfall, which affects total interceptional loss is a function of the rainfill type, as characterized by amount, duration and intensity. Also the rate of water removal, through evaporation, from the canopy upon the cessation of rainfall, is regulated by synoptic weather conditions.

Measurement of Interceptional Loss and Evaporation

Interceptional loss in this paper is treated as the amount of precipitation that is withheld from the ground by the aerial parts of vegetation. In terms of its conventional expression then, interceptional loss is measured by means of the following formula:*

$$I_L = P - (T_h + S_f). \tag{1}$$

Equation (1) is simply a measure of the difference in precipitation arriving at the top of the canopy and the forest floor. In a sense then, equation (1) views interceptional loss as a passive process in that it is implicitly assumed that intercepted water is evaporated away upon the cessation of rainfall. But interceptional loss is really a dynamic process in that evaporation of intercepted rainfall can also occur during the period of wetting and can very often exceed the intercepting capacity of the vegetation. As a result, several types of formulations have been suggested for measuring the total interceptional loss in a process oriented framework (Horton, 1919; Linsley, et al, 1949; Merriam, 1960; Rutter et al 1971/72, 1975).

* The symbols used are given as an appendix at the end of the paper.

In the present research, however, because of the uniqueness of the vegetal stand in question, both in terms of regional climate and canopy characteristics, new formulations must be developed. Also the emphasis here is not to describe the interception process, but rather to compartmentalize it so as to be able to determine the amount of evaporation in intercepted rainfall during and after the period of wetting.

Two techniques are utilized for measuring the total interceptional loss. The first method essentially makes use of equation (1). The second method assumes that all of the intercepted rainfall is essentially evaporated, thus allowing total interceptional loss to be calculated as total evaporative loss from the wetted canopy as:

$$LE_w = \frac{S(Rn - G) + \rho c \{e_s (T) - e\} / r_a}{S + \gamma} \quad . \qquad (2)$$

Equation (2) is essentially a variation of the combination model for estimating evaporation from wet surfaces and is attributable to Monteith (1965). The model is of practical applicability in the present context because it only requires measurements at a single height. This is a desirable condition for tall vegetation in view of the difficulty of setting up instruments at more than one level. Besides, the strong turbulent mixing generated by the rough forest canopy would preclude profile techniques on account of the need for very sensitive, but hazard-prone instrumentation.

If the interceptional loss were viewed to be a passive process (I_{LP}), in the sense that the amount of interception is only a function of the retention capacity, the following expression could be written:

$$I_{LP} = CS. \qquad (3)$$

It should be noted that equation (3) refers to canopy retention, or "residual storage" (Grah and Wilson, 1944) which could be equal to or less than the saturation value.

As suggested earlier, however, interceptional loss can in most instances be considered a dynamic process in that evaporation of intercepted water can occur during rainfall. Thus the dynamic interceptional loss (I_{LD}) can be found from:

$$I_{LD} = \int_{T_1}^{T_2} LE_w \, C\delta T \qquad (4)$$

where T_1 and T_2 are the times of beginning and ending respectively of the rainfall, and C is a constant for maintaining consistency of units. From equations (3) and (4) it follows that canopy detention storage (CS) can be calculated from:

$$CS = \int_{T_2}^{T_3} LE_w \, C\delta T \qquad (5)$$

where T_3 is the time of complete disappearance of intercepted rainfall from the canopy.

Finally, the total interceptional loss (I_{LT}) during and following each rain event can be calculated as:

$$I_{LT} = I_{LD} + CS \quad . \qquad (6)$$

Equation (6) can then be simplified to give:

$$I_{LT} = I_{LD} + \sum_{T = T_2}^{T_3} LE_w \qquad (7)$$

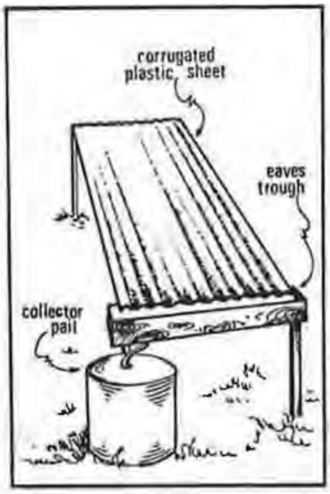
and finally, by considering equations (4) and (5), it can be reduced to:

$$I_{LT} = \sum_{T = T_1}^{T_3} LE_w \quad . \qquad (8)$$

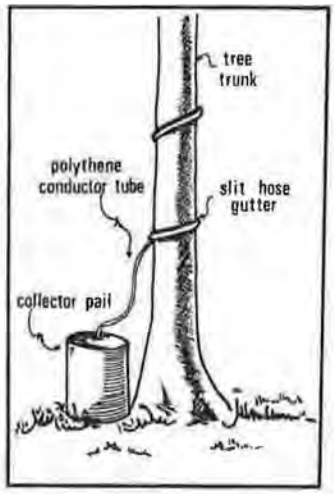
Site, Instrumentation and Experimental Procedure

1) Site

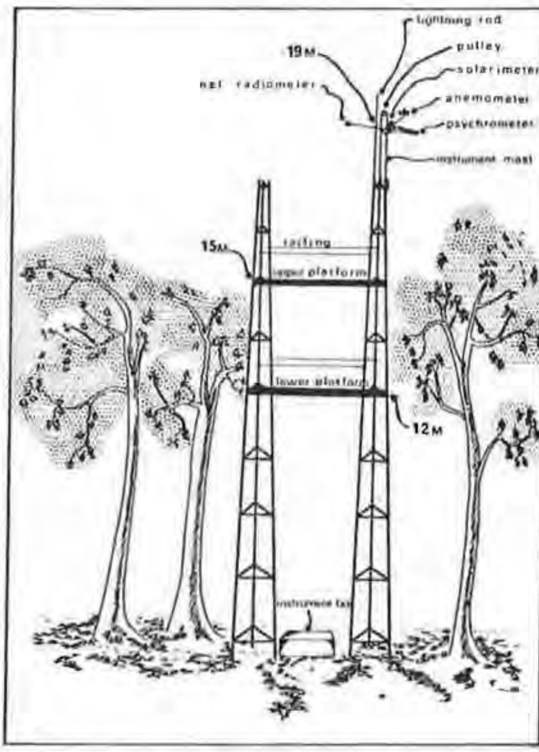
The field research for the experiment was conducted at Mont St Hilaire (45° 33' N, 73° 10' W) Quebec, about twenty miles east of Montreal. The mountain rises very sharply, in most places, from the St. Lawrence Lowlands up to a height of approximately 410 metres above sea level, which is about 370 metres above the surrounding plain. With the exception of a central lake, various steep rock surfaces on some of the outer slopes and an orchard in the interior of the basin, the mountain is completely forested. The forest consists of an undisturbed mixture of



(b) Throughfall Gauge



(b) Stemflow Gauge



(c) Tower Design and Instruments

Fig. 1. Instrumentation.

deciduous hardwoods, the dominant species being American beech (Fagus grandifolia Ehrh) and sugar maple (Acer saccharum Marsh). The average height of the trees is about 18 metres. The main experimental site was located on the southern gently sloping base of one of the hills. The period of field measurements spanned the growing seasons of 1974 and 1975.

2) Rain Measurements

Incoming precipitation in all forms over and within the forest, namely open, throughfall, stemflow and interceptional loss were measured. All measurements, except those for interception, were made directly, using different designs of gauges.

(i) Rain: Above Canopy. Because of the extreme difficulty in installing gauges above the canopy, two open sites were used to measure incoming precipitation. One gauge (12.7 cm diameter) was placed in an opening near the main experimental site. Appropriate exposure standards were satisfied by the location chosen. The other gauge was located in the open orchard where a climatological station was in operation. This was an M.S.C. design tipping-bucket rain gauge, with a 25.4 cm diameter receiver, and a recorder to keep track of rainfall intensities. Rain records were measured or checked following each rainfall.

(ii) Throughfall. Throughfall can be described as that portion of the rainfall which reaches the ground directly by penetrating the vegetative canopy through openings and as drip from leaves, twigs and stems. Throughfall was measured by means of six gauges located within the main experimental area. Because of the lengthy and arduous task of measuring large volumes of water by hand the number of gauges was kept at a minimum. The selection of gauge locations can best be described as systematic random, in that gauges were placed so as to sample the whole range of throughfall values: at least one gauge was placed where the canopy was thick, one where it was thin, and one where there was an opening. The type of gauge used was a table with a 1.2 m X 0.6 m corrugated plastic top that sloped at an angle, from the horizontal, of about 10 degrees, and drained into metal eaves troughs that in turn directed the water into plastic garbage pails with dimensions 0.6 m tall and 0.45 m upper diameter (Fig. 1a). A graduated plastic jug, calibrated against a standard measuring jar, and whose capacity was the equivalent of 12.7 cm of throughfall was used to facilitate measurements. As with rainfall measurements in the open, throughfall was measured after each period of rain.

(iii) Stemflow. According to Zinke (1967) stemflow can be described as that portion of the rainfall which, having been intercepted by the canopy, reaches the ground by running down the stems and branches and draining down the trunks. Stemflow was measured by taking a sample of trees that varied both in species (4 maple and 5 beech) and in trunk diameter. Again, sample size was restricted by the time-consuming task of measuring large volumes of water. The stemflow gauges consisted of ordinary (2.54 cm diameter) garden hose slit into two and wrapped around the trunks (Fig. 1b). The semihose was secured to the trunk with small tack nails and the remaining spaces were sealed with rubber chalk.

The lower end of the hose was left intact and drained into covered garbage pails that varied in dimension (0.60 m tall X 0.45 diameter to 0.91 m tall X 0.61 m diameter) according to tree size. The area of each stemflow gauge was derived by tracing the outline of the tree canopy as accurately as possible on the ground, and then estimating the area of the outline. This area was also calibrated against the area of a 12.7 cm diameter gauge and the same plastic jug as used for throughfall was used to facilitate measurements. Stemflow measurements were also done at the end of each rainfall.

3) Evaporation Measurements

(i) Meteorologic Parameters. As mentioned previously, evaporation from wetted leaves was calculated by means of equation (2). Since average hourly measurements were required for the purposes of this study, continuous records of net radiation, soil heat flux, wind speed, and air temperature and humidity were obtained, at least for the period of daylight.

Except for soil heat flux, all variables were measured at a reference height above the canopy. To achieve this end two triangular television towers, consisting of 2.5 metre steel sections and rungs at every 0.7 metre, were constructed 2.5 metres apart (Fig. 1c). These towers were secured by guy wires attached to neighbouring trees and protruded to a height of about a half-metre above the forest top. A 3-metre mast with instrumentation for measuring above-canopy parameters, namely net radiation, dry and wet-bult air temperatures, and wind speed, was then attached to one of the towers, with its top at a reference height of about 1.5 metres above the mean height of the canopy.

Net radiation was measured by means of an S.R.I. net radiometer that had a constant calibration factor of 55.8 mv/ly min $^{-1}$. The sensing element was on a long enough arm so as to avoid tower interferences. In order to equalize convective heat losses from both sides of the thermopile plate the polyethylene domes were kept inflated by means of an aquarium pump housed at the bottom of the tower. The air was however first blown past a reservoir of silica gel so as to prevent internal condensation. The proper pressure adjustment of the pump was made through manipulation of the bubble rate (4-5 per minute) in a water bottle, into which the back pressure was fed. A pair of home-made copper constantan thermopile flux plates calibrated against a commercially manufactured (Middleton and Co.) instrument were used to measure the flux of soil heat. These plates were placed at a depth of about 5 cm adjacent to the tower site and were connected in series thereby giving an average sensitivity of 1 mv/0.07 ly min $^{-1}$.

The dry and wet bulb temperatures of the air, which were also necessary for humidity calculations, were measured at reference height (1.5 m above canopy) by means of home-made copper-constantan thermocouples that have a calibrated sensitivity of 39.5 $\mu v/°c^{-1}$. The thermocouples were shielded and insulated by 2 sizes of P.V.C. pipes, the outer pipe being coated with aluminum foil to restrict radiation absorption. Artificial ventillation was used to aspirate the thermocouples by drawing air past the bulbs at a rate of about 4 metres sec^{-1}, with a vacuum fan.

The aerodynamic resistance (r_a) to vapour diffusion was measured according to the following formulation:

$$r_a = \frac{1}{K^2 U} \left(\ln \frac{Z - d}{Zo}\right)^2 \qquad (9)$$

Wind speeds were measured at reference height by means of a Cassela Type (16108/1 sensitive aremometer with a 3-cup metal rotor. The method of obtaining plant parameters, namely the zero plane displacement (d) and the roughness length (Zo) is discussed elsewhere (Singh, 1976).

4) Supporting Measurements

(i) Duration of Wetness. In order to delineate the periods when intercepted water was present on leaf surfaces, a moisture sensor designed by St. Laurent (1973) was utilized. The instrument is an excellent device and provides an objective method for detecting the presence of water on a leaf surface. The principle of operation is that whenever there is moisture present between the pair of probe tips consisting of high conductance wire, and connected to a pair of resistors, a low-level positive voltage passes through the two resistors to the gate of a Silicon controlled rectifier (SCR); which, when fired, triggers an alarm or light signal. To avoid having to reset the sensor every time the SCR was fired, a stepped-down AC voltage (12 volt) was used, utilizing a miniature transformer. The power leads were then hitched to a light bulb fitted to a rectifier to obtain DC output. This was in turn connected to an I.C.A. model 400 recorder via a voltage divider so that when the sensor was triggered a deflection was recorded. In view of the fact that the different levels of the canopy dried out at different times, 2 sensors were used. One was placed at the top of the canopy and the other within the canopy.

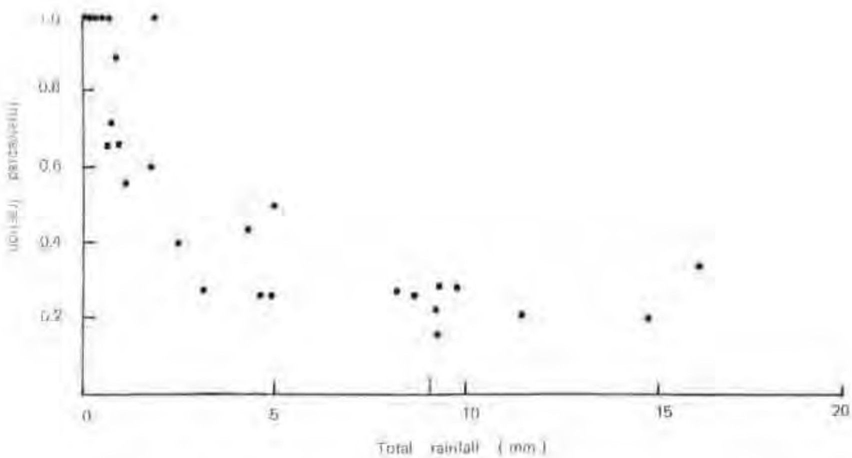


Fig. 2. Intercepted fraction vs total rainfall, August 1 to October 2, 1974.

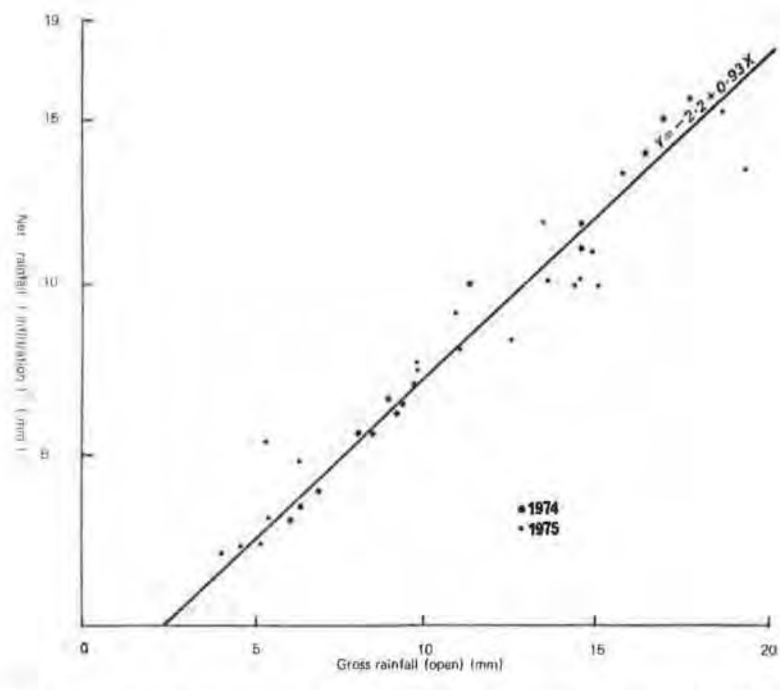


Fig. 3. Determination of storage capacity (2.4 mm) of canopy.

(ii) Evapotranspiration: Dry Periods. Since it was necessary to differentiate between a wetted and a dry canopy, evapotranspiration estimates were measured by means of the following formulation:

$$LE_d = \frac{S(Rn - G) + \rho c \{ e_s (T) - e \} / r_a}{S + \gamma (1 + r_c/r_a)} \quad . \qquad (10)$$

Equation (10) is due to Monteith (1965), and is designed to measure latent heat transfer for non-potential surfaces, as characterized by a finite canopy resistance (r_c). The method for deriving regular estimates of the parameter r_c is discussed elsewhere (Singh, 1977).

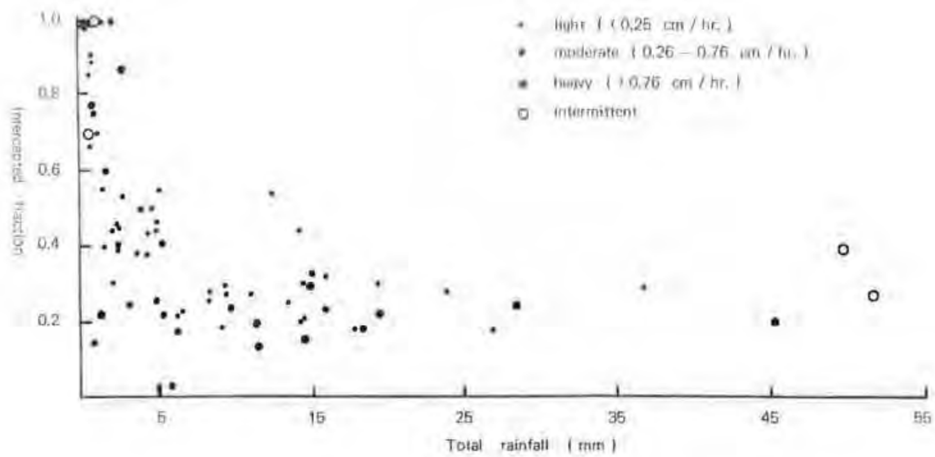
Discussion of Results

1) Evaporation During Rainfall

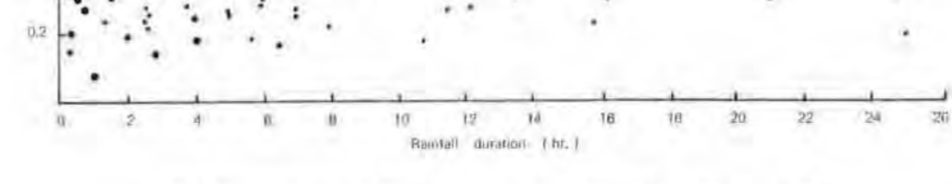
Since evaporation of intercepted rainfall can often occur during the period of wetting, it follows that the amount of interceptional loss depends not only on the intercepting capacity of the forest stand, but also on the amount of concurrent evaporation from the canopy during the period of rainfall.

Reference to Figure 2 shows that, on the average, the fraction of total precipitation intercepted is about 20 to 30 percent for rainfill amounts greater than about 5 mm. Similar percentages were observed by Horton (1919), Zinke (1967), Rogerson and Byrnes (1968), and Bultot et al (1972), for comparable vegetation types. For decreasingly lesser amounts (< 5 mm) the intercepted fraction increases exponentially and a saturation point, at which almost all of the precipitation is intercepted, is reached, at rainfall amounts of about 2 mm and less. This would suggest that the intercepting capacity of the vegetal cover being discussed is in the vicinity of 2 mm of water. It must be observed that some throughfall can occur, before saturation of the intercepting capacity is reached, since some raindrops can penetrate the canopy through open spaces, or can splash off the edges of the leaves.

In order to delimit the intercepting capacity (passive) of the canopy, gross precipitation (above canopy) was plotted against net precipitation (throughfall plus stemflow). The data points used are for medium and intense rainfalls (> 26 cm/hr) of short duration (less than a couple of hours), so as to subdue the effect of evaporation, if any, during rainfall. Also only rainfall amounts greater than 4 mm were used,



(a) Function of Amount and Intensity 1974, 1975.



(b) Function of Duration and Intensity 1974, 1975.

Fig. 4. Intercepted Fraction of Rainfall.

since as observed in Figure 2, and as noted in previous experiments (Horton, 1919; Rowe and Hendrix, 1951; Leyton et al, 1967), the slope of the regression of net against gross precipitation changes because net rainfall approaches zero beneath this approximate critical value. The intercept of the best-fit line with the gross precipitation axis gives an estimate of the intercepting capacity of the canopy, which in this case happens to be 2.4 mm (Fig. 3). This value is similar to that observed by Zinke (1967) and Bultot et al (1972), for a similar type of vegetation. It must be remembered however, that the intercepting capacity can vary, since wind-induced shaking of the branches, which causes mechanical removal of water from the leaf surfaces, can cause it to be lower than its maximum value.

Since there exists a range of values for the fraction of rainfall intercepted, even for rainfall amounts greater than the intercepting capacity (Fig. 2), evaporative losses as measured by equation (4) during the period of wetting must be critical. Evaporative loss during rainfall on the other hand, can be related to rainfall types as characterized by amount, intensity and duration. Rainfalls for both growing seasons (1974 and 1975) were grouped into 3 intensity categories following the classification of the Atmospheric Environment Service of Canada, as set out in MANOBS (1961): light (< 0.25 cm/hr), moderate (0.26 to 0.76 cm/hr) and heavy (>0.76 cm/hr).

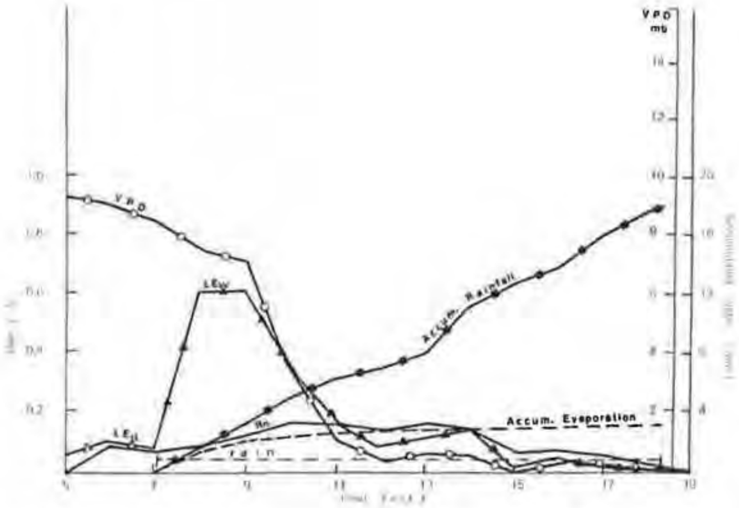
All rainfall amounts less or nearly less than the canopy storage capacity (2.4 mm) can be expected to be intercepted, except in cases where excessive windiness reduces the retention capacity of the leaves. For rainfall amounts greater than the storage capacity of the canopy however, intensity and duration of rainfall can affect interceptional loss through their regulating effect on evaporation during wetting.

When precipitation totals exceed the intercepting capacity light intensity rainfalls usually have a greater proportion of their total amounts intercepted than moderate and heavy intensity rainfalls (Fig. 4a). An examination of Figure 5 a shows that for a very light rain that fell throughout the day, evaporation of intercepted rainfall occurred throughout the period of wetting. Saturation of the ambient air actually never took place because of windy conditions that provided continuous ventilation. Over a period of about 11 hours an accumulated total of 3 mm of evaporation occurred. Most of this amount was accounted for during the initial stages of the rain, where the saturation deficit of the air was still relatively high. Rainfalls with the greatest duration, therefore, which is typical of light intensity showers, generally have a greater portion of their totals intercepted, provided that the ambient air is not saturated with moisture (Fig. 4b). The interceptional fraction is even greater for very light intermittent showers in that under these circumstances not only was the canopy allowed to dry out partially, but also the evaporating power of the air, as characterized mainly by its saturation deficit, was successively increased, upon temporary cessation of wetting. On some occasions, however, light intensity rainfalls were preceded by extended cloudy and humid conditions. As a result by the time the rain commenced, the vapour pressure deficit of the ambient air was already quite low and reached saturation in a short period of time, thereby restricting substantial evaporative losses and, hence interceptional loss during the rainfall. Also, in exceptional cases, where separate showers were closely spaced, the canopy sometimes failed to dry out completely before the onset of the next rainfall. In these instances most of the subsequent rain reached the ground since canopy storage was already satisfied by the previous rainfall, thereby reducing the intercepted fraction.

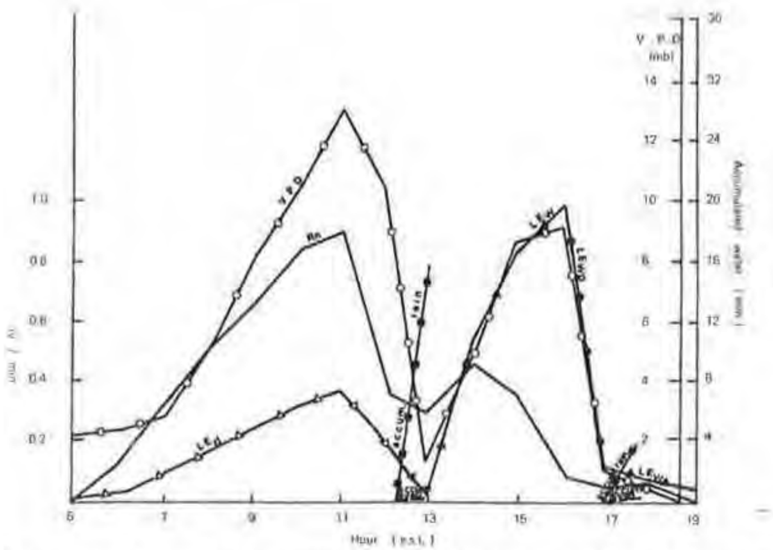
During rainfalls of medium and heavy intensities, evaporation of intercepted rainfill, was generally found to be minimal, being at or near the intercepting capacity of the canopy (Fig. 4a). Under these conditions, saturation or near-saturation of the ambient air was quickly obtained. The rather short durations of these showers also inhibited sizeable evaporative losses during wetting (Fig. 5b). In exceptional cases, however, a small amount of evaporative loss was observed to occur during the initial stages of these rainfall types. These occasions occurred either when the rainfall arrived suddenly, thus allowing the saturation deficit of the ambient air to be high initially, or when extremely windy conditions advected unsaturated air, at least during the early stages of the rainfall.

2) Effect of Post-Wetting Weather

Post-wetting synoptic conditions dictate the rate at which canopy detention storage is depleated. This rate can be measured by means of equation (5). Upon the cessation of rainfall both the input of incoming solar energy and the vapour pressure deficit of the ambient air



(a) During Light Intensity Precipitation, May 30, 1975.



(b) During Heavy Intensity Precipitation, July 19, 1975.

Fig. 5. Evaporation of Intercepted Water.

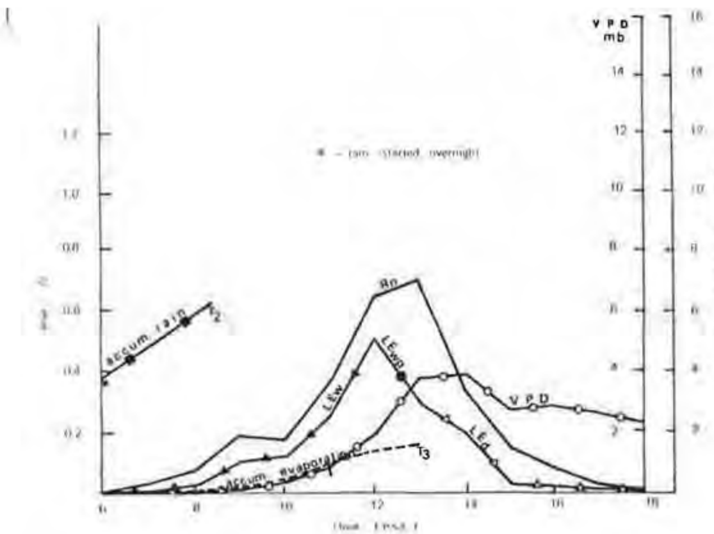


Fig. 6. Increase in evaporation of intercepted water upon cessation of rainfall, September 17, 1975.

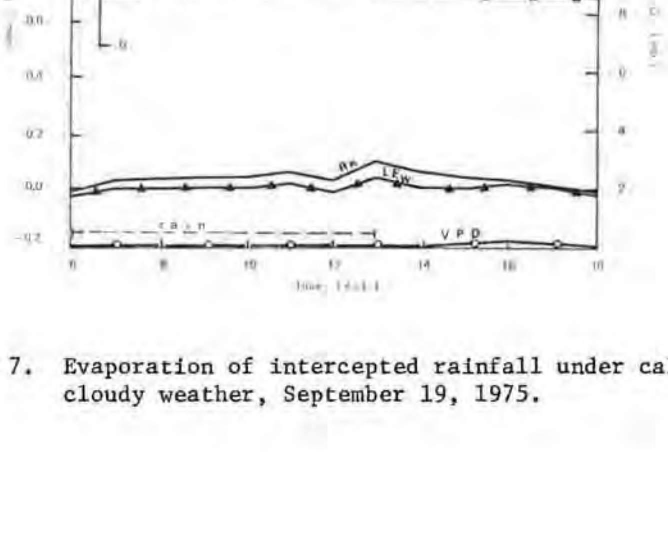


Fig. 7. Evaporation of intercepted rainfall under calm, cloudy weather, September 19, 1975.

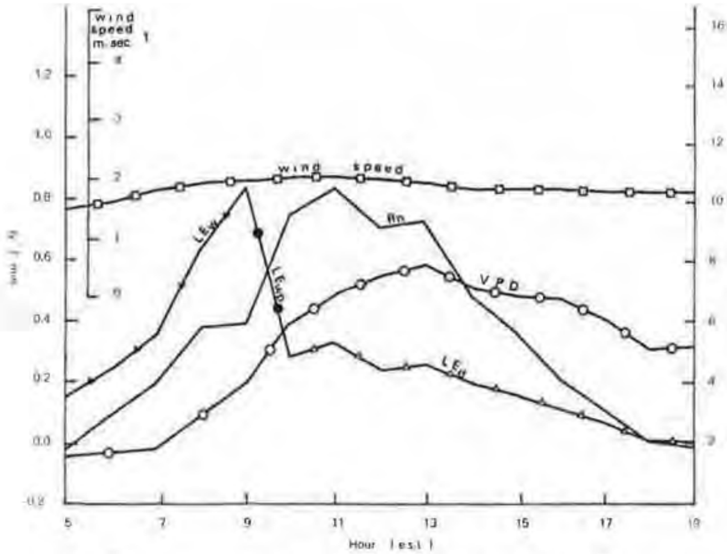
normally increase, at least for the daylight period (Fig. 6). Clearing following rainfall in the study area was, however, found to be rather slow, in that cloudy conditions prevailed, following wetting, for extended periods in the majority of instances. The vapour pressure deficit of the ambient air, especially when windy conditions continually advected warmer and drier air, was therefore found to be more critical than radiation receipt in controlling post-wetting evaporative losses.

An examination of Figure 6 shows that the evaporation of intercepted rainfall following wetting proceeds at a faster rate than during rainfall. The amount of water to be evaporated is usually of the same order of magnitude as the storage capacity of the vegetation. This intercepted moisture can however be evaporated in from about one to several hours upon the cessation of rainfall (Fig. 7). As a result, because of limited energy receipt and restricted ventilation, the evaporation of intercepted rainfall proceeded at a rather slow rate, so that the canopy remained wet for an extended period of time. On other occasions, however, especially when the rain had fallen overnight or early morning, relatively clear skies together with unsaturated ambient air followed wetting of the canopy (Fig. 8a). On these occasions, drying of the canopy was achieved in a relatively short time period, especially when wind conditions provided a continuous supply of moderately dry air.

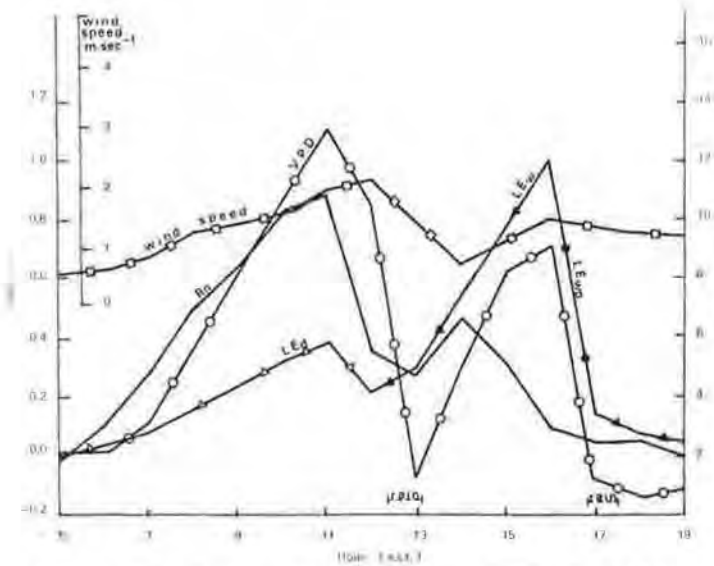
It follows from the above then that timing of the cessation of wetting is also important. When rain cessation occurred during the early morning or early afternoon, the canopy usually dried out before nightfall, the same day (Fig. 8a and 8b), but when the rainfall stopped during the early evening or overnight (Fig. 8b), in most cases it was not until the following day that the canopy became dry. The greater amount of overnight rains were usually preserved on the canopy until the next morning, except in cases where strong winds advected fair amounts of energy for latent heat transfer.

3) Comparison of Estimates

In order to asses the accuracy of the estimated amount of latent heat exchange during wet periods, the total amount of evaporation of intercepted water during and after each rainfall, as calculated by equation (8) is compared with the amount of interceptional loss, measured



(a) Sunny Day, Following Overnight Rainfall,
June 15, 1975.



(b) Early Afternoon and Evening, July 19, 1975.

Fig. 8. Evaporation of intercepted rainfall.

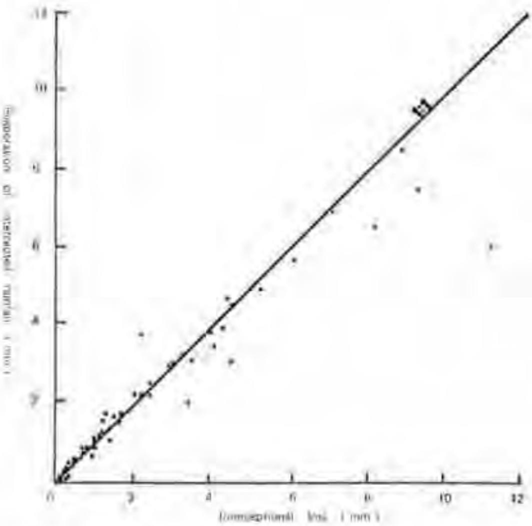


Fig. 9. Evaporation of intercepted rainfall vs measured interceptional loss, 1975.

in terms of equation (1) (Fig. 9). It is readily noticeable that except for the higher values of evaporation or interceptional loss (i.e.> 10 mm), there exists a close correspondence between estimated evaporation of intercepted rainfall and measured interceptional loss. The discrepancy for the higher values is mainly attributable to the fact that evaporative losses were not measured during the night. As remarked by Penman (1963), there can occur a substantial amount of evaporation at night, in the presence of a pronounced vapour pressure deficit of the ambient air and a relatively low aerodynamic resistance such as occurs during periods of strong windiness. As a result, if night-time conditions are omitted, there will be a significant underestimation of the evaporation of intercepted rainfall. This effect was especially common when the forest remained wet at night, following a late afternoon or early evening shower. Other inconsistencies arose mainly from measurement errors for both variables, especially for the evaporation of intercepted rainfall, where the delimitation of a wet or partially wet, as opposed to a dry canopy, was somewhat imprecise.

Also, since the intercepting capacity of the canopy is about 2.4 mm, it is evident from Figure 9 that in some cases anywhere from 6 to

8 mm of water could be lost through dynamic interception loss, i.e. through evaporation during rainfall. These limited but extremely high values, however, were derived during periods of light and prolonged or intermittent showers, when the saturation deficit of the ambient air was always relatively high.

4) Effect on Water Balance

The effect of rainfall characteristics and post-wetting weather conditions is critical to the nature and magnitude of intercepted rainfall. As mentioned previously, the type of rainfall can regulate the amount of dynamic interceptional loss (I_{LP}), which constitutes a direct loss of moisture to the soil. Also the length of the period that the canopy remains wet, as dictated by rainfall duration and post-wetting weather, has important implications in terms of water consumption by vegetation. If, as observed by Jones (1957), the transpirational withdrawal of soil moisture is subdued during periods when leaves are wetted, then intercepted rainfall contributes indirectly to soil water.

In instances where the canopy remained wet for extended periods, as during and following light intensity rains (Figs. 5a and 7), there can occur a substantial amount of transpirational saving by the canopy. At the same time, however, it must be remembered that there can occur a fair amount of evaporation during this type of rainfall (Fig. 5a). Thus, although some soil moisture is conserved, a less proportion of the total rainfall reaches the ground. On the other hand, for short-lasting, moderate and heavy intensity rainfalls (Fig. 5b), there is limited time for evaporation of intercepted water during rainfall. Consequently, most of the precipitation reaches the ground provided that the rainfall amount is greater than the canopy storage capacity.

Also, depending on post-wetting weather, the duration of soil moisture saving can last from one to several hours. On occasions when the intercepted rain is quickly evaporated (Fig. 6), there is little saving of soil water. But when the canopy remains wet for an extended time period (Fig. 7) there can arise a substantial amount of daily transpirational saving. Short, heavy intensity rains followed by a period of restricted evaporation may therefore be the most conducive to the conservation of soil water, in that they not only provide a greater supply of soil moisture, but also subdue transpirational withdrawal for an extended period. In the case of overnight and late evening rains, although evaporation of intercepted rainfall is restricted, there is nevertheless very little transpirational saving, in that stomates are usually closed at night.

In summary, then, it can be stated that rainfall characteristics, namely amount, duration, intensity and time of occurrence, regulate the amount of evaporation during wetting and hence total interceptional loss. The rate and efficiency of water loss from a wetted canopy is also controlled by post-wetting weather. Both of these factors in turn affect soil moisture consumption and hence the computation of the water balance.

LIST OF SYMBOLS

I_L	interceptional loss (mm).
P	gross rainfall - above canopy (mm).
T_h	throughfall (mm).
S_f	stemflow (mm).
I	interceptional loss (mm), passive (LP), dynamic (LD), total (LT).
S	slope of the saturation vapour pressure curve at air temperature (T) (mbar $°c^{-1}$).
ρc	volumetric heat capacity of dry air (2.9×10^{-4} cal. cm^{-3} $°c^{-1}$).
γ	psychrometric constant (0.66 mbar $°c^{-1}$).
e, es	actual or saturation vapour pressure of air temperature (T) at reference height (mbar).
r_a	aerodynamic resistance to vapour transfer at reference height (secs. cm^{-1}).
r_c	canopy resistance to vapour diffusion (secs. cm^{-1}).
Rn	net radiation (cal. cm^{-2} sec. $^{-1}$).
G	soil heat flux (cal. cm^{-2} cec. $^{-1}$).
LE_w	latent heat of evaporation from a wet canopy (cal. cm^{-2} sec. $^{-1}$).
LE_d	latent heat of evapotranspiration (cal. cm^{-2} sec. $^{-1}$).
V.P.D.	vapour pressure deficit of the ambient air (mbar).
CS	canopy retention storage (mm).
K	vonKarman's constant.
d	zero plane displacement (cm).
Zo	roughness length (cm).
Z	reference height (cm).

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THE DERIVATION OF A FORMULA TO CALCULATE DIRECT SHORT-WAVE RADIATION ON SLOPES

by

Atsumu Ohmura*

The instantaneous intensity of direct shortwave radiation on any slope is a composite of the energy being delivered by the sun's rays and its modification by the angle and azimuth of the slope in question. The monochromatic intensity $I\lambda$ of the solar beam at the surface is given by

$$I\lambda = I\lambda o \rho\lambda^{m} \quad (1)$$

where $I\lambda o$ is the monochromatic intensity of the solar beam at the top of the atmosphere, $\rho\lambda$ the monochromatic zenith-path transmissivity, and m the optical air mass.

The integration of equation (1) with respect to wavelength provides the total incident intensity at the ground surface. This integration permits the derivation of a simple law of transmission (Haltiner and Martin, 1957) which, if used to denote the direct solar energy falling on 1 cm^2 for 1 min, can be conveniently written as

$$Im = Io\rho^{m}, \quad (2)$$

where Im is the direct solar radiation (ly min^{-1}) reaching a surface normal to the sun's rays, Io is the intensity of solar energy at the top of the atmosphere (given by the solar constant divided by the square of the radius vector of the earth), and ρ is the mean-zenith-path transmissivity of the atmosphere.

The modification to Im introduced by a given slope is a function of the relation between the angle and azimuth of the slope on the one hand,

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and the azimuth and height of the sun on the other. If these respective characteristics of slope and sun are expressed by unit coordinate vectors, the flux per minute, Is, of direct shortwave radiation on a given slope is the product of Im and the cosine of the angle between the two vectors, such that

$$Is = Io\rho^{m} \cos(\vec{X} \wedge \vec{S}), \quad (3)$$

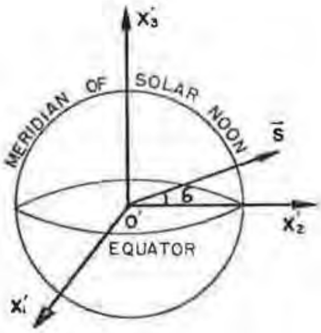
where $\vec{X}$ is a unit coordinate vector normal to the slope and pointing away from the ground, $\vec{S}$ a unit coordinate vector expressing the height and position of the sun, and $\wedge$ the symbol denoting the angle between $\vec{X}$ and $\vec{S}$.

The computation of direct short-wave radiation on slopes by equation (3) is possible only if the equation is solved as a function of time. This procedure is simplified by treating the sun as moving north and south with the seasons along the plane of the meridian on solar noon for the site, and by treating the slope as rotating about the earth's polar axis once in twenty-four hours.

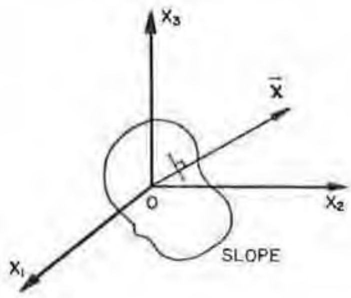
The direction of the sun can be expressed in a normal coordinate system of which the three axes are illustrated in Figure 1a. One axis consists of the earth's polar axis, a second is parallel to the plane of the equator cutting the meridian of solar noon as shown in the figure, and the third is directing normal to the other two axes. For convenience of description, this system will be called a global system.

A coordinate system which can conveniently express $\vec{X}$ and also be easily correlated to the global system is illustrated in Figure 1b. This system will be called a local system since it is located at the site. Its axes are directed to zenith, south and east as is shown in Figure 1b. Figure 1c shows the mutual relation between the global and the local system.

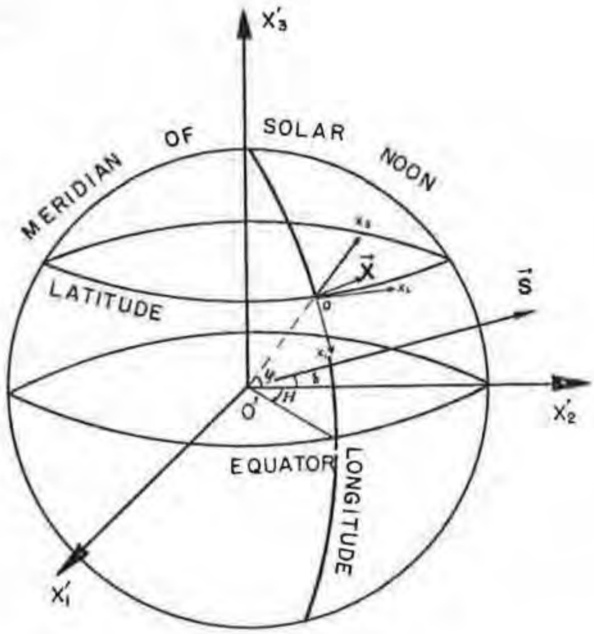
To complete the solution of equation 3, the vectors $\vec{X}$ and $\vec{S}$ must be discussed in a common system. It is mathematically equally possible either to transfer the local system into the global system or the global system into the local system. For this study, the former procedure has been adopted, so that the local system is regarded to be rotating once a day around the polar axis in the declination of $90° - \zeta$, where ζ is the latitude of the place.



(a) Global System



(b) Local System



(c) Relationship between Global and Local Systems

Fig. 1. Co-ordinate systems for radiation calculations.

In the local system, vextor $\vec{X}$ can be expressed in the following way (Fig. 1b):

$$\vec{X} = (x_1, x_2, x_3) \tag{4}$$

where x_1 is -cosA sinB, x_2 sinA sinB, and x_3 cosB.

A is the azimuth of the slope measured from north with positive value to the east. B is the zenith angle of vector $\vec{X}$. The expression of vector $\vec{S}$ in the global system (Fig. 1a) is:

$$\vec{S} = (0, \cos\delta, \sin\delta) \tag{5}$$

where δ is the sun's declination, positive when the sun is north of the equator. The relationship between the global and the local system is illustrated in Figure 1c.

To transfer vector $\vec{X}$ to the global system, new components x_1', x_2', x_3' will define vector $\vec{X}$ as follows:

$$\vec{X} = (x_1', x_2', x_3') \tag{6}$$

The theorem of vector algebra enables the old components to be related to the new components, such that

$$\vec{X} = (x_1', x_2', x_3') = \begin{vmatrix} t_{11} & t_{21} & t_{31} \\ t_{12} & t_{22} & t_{32} \\ t_{13} & t_{23} & t_{33} \end{vmatrix} \quad (x_1, x_2, x_3) \tag{7}$$

where $t_{11} = \cos(\vec{X}_1 \Lambda \vec{X}_1')$, $t_{12} = \cos(\vec{X}_1 \Lambda \vec{X}_2')$, $t_{13} = \cos(\vec{X}_1 \Lambda \vec{X}_3')$, $t_{21} = \cos(\vec{X}_2 \Lambda \vec{X}_1')$, $t_{22} = \cos(\vec{X}_2 \Lambda \vec{X}_2')$, $t_{23} = \cos(\vec{X}_2 \Lambda \vec{X}_3')$, $t_{31} = \cos(\vec{X}_3 \Lambda \vec{X}_1')$, $t_{32} = \cos(\vec{X}_3 \Lambda \vec{X}_2')$, and $t_{33} = \cos(\vec{X}_3 \Lambda \vec{X}_3')$; where $\vec{X}_n$ and $\vec{X}_n'$ (n = 1, 2, 3) are the vectors to define axes $\vec{X}_n$ and $\vec{X}_n'$. Using the definition of the scalar product of two vectors, it follows that:

$$\cos(\vec{X} \Lambda \vec{S}) = \frac{\vec{X} \cdot \vec{S}}{|\vec{X}| \cdot |\vec{S}|}$$

Since $\vec{X} = 1$, $\vec{S} = 1$,

$$\begin{aligned} \cos(\vec{X} \Lambda \vec{S}) &= |\vec{X}| \cdot |\vec{S}| \\ &= (x_1', x_2', x_3')(0, \cos\delta, \sin\delta) \end{aligned}$$

and from Equation (7)

$$\cos(\vec{X} \Lambda \vec{S}) = \begin{vmatrix} t_{11} & t_{21} & t_{31} \\ t_{12} & t_{22} & t_{32} \\ t_{13} & t_{23} & t_{33} \end{vmatrix} \quad (x_1, x_2, x_3)(0, \cos\delta, \sin\delta)$$

$$= (t_{12}x_1 + t_{22}x_2 + t_{32}x_3)\cos\delta + (t_{13}x_1 + t_{23}x_2 + t_{32}x_3)\sin\delta \tag{8}$$

The next step is to specify the six direction cosines in equation (8) in terms of the latitude of the place (ζ) and the hour angle (H). Reference of Figure 1c, showing the relation between the two coordinate systems, indicates that the direction cosines are:

$$t_{12} = \sin\zeta \cos H,\ t_{22} = -\sin H,\ t_{32} = \cos\zeta \cos H,$$
$$t_{13} = -\cos\zeta,\qquad t_{23} = 0,\qquad t_{33} = \sin\zeta.$$

Thus equation (8) becomes

$$\cos(\vec{X}\Lambda\vec{S}) = (x_1 \sin\zeta \cos H - x_2 \sin H + x_3 \cos\zeta \cos H) \cos\delta + (-x_1 \cos\zeta + x_3 \sin\zeta) \sin\delta \qquad (9)$$

For convenience, it is desirable to express the optical air-mass (m) in terms of the elements used to express $\vec{X}$ and $\vec{S}$. This can be done by means of the secant approximation for m, which can be used when the solar zenith angle is smaller that 70° (List, 1966). If z is solar zenith angle, by the secant approximation for m,

$$\begin{aligned} m &= \sec z \\ &= \frac{1}{\cos z} \\ &= \frac{1}{\cos(\vec{X}\Lambda\vec{S})} \end{aligned} \qquad (10)$$

where $\vec{X} = (0, 0, 1)$ and it is assumed that $z \neq 90°$. From equation (9),

$$m = \frac{1}{\cos\zeta \cos\delta \cos H + \sin\zeta \sin\delta} \qquad (11)$$

Applying equations (9) and (11) to equation (3),

$$I = \frac{I_o}{r^2} P_{(1)}^{\ 1/(\cos\zeta \cos\delta \cos H + \sin\zeta \sin\delta)} (x_1 \sin\zeta \cos H - x_2 \sin H + x_3 \cos\zeta \cos H) \cos\delta + (-x_1 \cos\zeta + x_3 \sin\zeta) \sin\delta \qquad (12)$$

This is the formula to calculate the flux of direct short-wave radiation on any surface of the earth.

To use equation (12) for the calculation of incident energy for a certain time period, it must be integrated with respect to time. Although the mathematical integration of Equation (12) for time is not possible, it can be achieved to sufficient accuracy by means of a summation-approximation in which a sufficiently small time interval is used.

For daily totals, the integration then becomes,

$$I_d = \sum_{H=H_1}^{H_2} I \,\Delta H \tag{13}$$

where ΔH is a sufficiently small time interval. A time interval of twenty minutes (ΔH = 20 minutes) has been found to be suitable for this purpose (Garnier and Ohmura, 1970).

To perform the computation of Equation (13) it is necessary to know H_1 and H_2.

If the topography around the site of interest does not cast a shadow to the place, or if the influence of the shadow is negligibly small, H_1 and H_2 can be determined by the times of the astronomical sunrise and sunset together with the times when the sun can shine for the first time and the last time in a day on the slope which is assumed not to experience any shadow effect from the existence of the earth. By "time of sunrise and sunset", is meant the moments when the centre of the sun reaches the horizon or the plane of the surface of interest.

At the astronomical sunrise and sunset, $\vec{X}\wedge\vec{S} = 90°$, where $\vec{X} = (0, 0, 1)$. Then, equation (9) becomes,

$$\cos\zeta \cos\delta \cos H + \sin\zeta \sin\delta = 0$$

from which it follows that

$$H = \arccos(-\tan\zeta \tan\delta). \tag{14}$$

Among the possible values for H, the two smallest in absolute number will indicate either sunrise or sunset. There are two such possible numbers, of which that having a negative sign will indicate the time of sunrise and that having positive sign will indicate that of sunset.

To obtain the times of sunrise and sunset for a slope of a given azimuth and gradient, it is necessary to imagine a slope which will never be in shadow by any obstacles except its own existence.

At the moment when sunrise or sunset takes place on such a slope, $\vec{X}\wedge\vec{S} = 90°$. Equation (9) then becomes,

$$(x_1 \sin\zeta \cos\delta + x_3 \cos\zeta \cos\delta) \cos H - x_2 \cos\delta \sin H = -(-x_1 \cos\zeta + x_3 \sin\zeta) \sin\delta.$$

Solving this equation for H,

$$C \sin(H + \alpha) = -(-x_1 \cos\zeta + x_3 \sin\zeta) \sin\delta$$

where

$$C = \sqrt{(x_1 \sin\zeta \cos\delta + x_3 \cos\zeta \cos\delta) + (x_2 \cos\delta)^2}$$

$$= \arctan \frac{-x_2 \cos\delta}{x_1 \sin\zeta \cos\delta + x_3 \cos\zeta \cos\delta}$$

$$= \arctan \frac{-x_2}{x \sin\zeta + x \cos\zeta}$$

Therefore,

$$H + \alpha = \arctan \frac{-(-x_1 \cos\zeta + x_3 \sin\zeta) \sin\delta}{M}$$

where

$$M = \sqrt{H^2 - (-x_1 \cos\zeta + x_3 \sin\zeta)^2 \sin^2\delta}$$

Finally,

$$H = \arctan \frac{-(-x_1 \cos\zeta + x_3 \sin\zeta) \sin\delta}{M} - \alpha \qquad (15)$$

A comparison of the solution of Equations (14) and (15) will yield the following conclusions from which the values of H_1 and H_2 in equation (13) can be obtained. The sunrise and sunset from equation (14) and (15) are H_{a1}, H_{a2} and H_{s1}, H_{s2} respectively. If the astronomical sunrise is the true sunrise. This can be written in the following way:

	If $H_{a1} > H_{s1}$	then	$H_1 = H_{a1}$
Similarly,	if $H_{a1} \leqq H_{s1}$	then	$H_1 = H_{s1}$
	if $H_{a2} > H_{s2}$	then	$H_2 = H_{s2}$
	if $H_{a2} \leqq H_{s2}$	then	$H_2 = H_{a2}$.

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NEWS AND COMMENTS

The following report is contributed by John Lewis of McGill University, who was the "Climatologist in Orbit" for the 1976-77 season.

"When I was informed of the opportunity to be the "Orbiting Climatologist", I sprang at the invitation. Being new on the Canadian scene, the role of the "Orbiting Climatologist" offered me the chance to visit and meet some of my Canadian colleagues while at the same time spreading the research word concerning my approach to urban climatology.

The topics I suggested for the various presentations were 1) a general methodological treatment of urban climatology, 2) urban energy budget modelling, and 3) land use effects and urban air pollution modelling.

I started my term the first part of November, 1976 by travelling to the University of Western Ontario and the University of Windsor. On November 3rd I spoke to a 2nd year physical geography class, after which I spent a pleasant day with Bob Packer, Roger King, and Brian Luckman. The next day I proceeded to the University of Windsor where I spoke to John Jacobs' climatology class. During these two days I was competing with Bill Bunge's travelling 'road show'. These two lectures, in many ways, posed as an interesting comparison for the students. The first week in December I ventured once again to the University of Waterloo to visit with Geoff McBoyle and speak to his 3rd year physical geography-climatology class.

The next part of the adventure was a three day trip around the southern shores of Lake Ontario visiting McMaster, University of Toronto-Erindale, and Atmospheric Environment Service, Downsview at the end of January. I presented two lectures at McMaster University, one to Hank Hannel's 4th year climatology course, the other to a seminar group of graduate students and faculty. It should be said that John Davies and Wayne Rouse's hospitality was unsurpassable. The next day was spent at the University of Toronto with Scott Munro touring the facilities of this new campus. An evening presentation was given to Scott's 3rd year climatology class. Scott advertized the talk widely and as a result, people from five or six other academic disciplines attended. A rather lengthy but lively discussion occurred after the lecture and at the end of the evening we were even interviewed by a reporter from a local paper. The third day I journeyed a short way to the Atmospheric Environment Service to present a seminar sponsored by Morley Thomas, Gordon McKay, and Dave Phillips. This was the day of the famous Toronto-Ontario snow storm of '77. It took one hour to travel three miles before I arrived at AES Friday morning, and thirteen hours starting Friday afternoon to return from Toronto to Montreal by train. Climatologists are always searching for predictable indicators. Singularities are supposedly a type of indicator. Posterity may pass me by but I propose the Lewis-AES singularity because I have been to AES twice since coming to McGill and both times they have closed because of the weather. The first time, the famous storm at the beginning of April, 1975 when the members of AES were snowed in literally; and this past storm in January. My skill score is still good!!

The last trip was to Queen's University for a two day visit with Harry McCaughey and the Geography Department. I spoke to a 2nd year climatology class and on the second day a lecture was given to a 3rd-4th year climatology group. My two day stay afforded me a lot of time for discussion and exchange of ideas with Harry and his colleagues. A very pleasant time.

In between these last two journeys, I literally walked down the street (Sherbrooke Street in Montreal) to visit Dave Frost at Concordia University. There I spoke to his climatology class, after which followed a very interesting discussion concerning land use modification in Montreal and its effect on urban climate.

As my predecessors have stated, I wish to reiterate strongly my support for the continuation of the 'Orbiting Climatologist'. It certainly was a valuable experience for me. I hope it was for my audience too.

Finally, I would like to thank Ben Garnier for giving his support during my tenure as 'Orbiting Climatologist'."

The search for new and renewable energy sources provides climatologists with many opportunities to develop work in applied climatology, especially in respect to solar energy and wind energy. The Solar Energy Society of Canada is actively engaged in promoting work in these fields. In August, 1976, the Society hosted a major meeting in Winnipeg in conjunction with the American Section of the International Solar Energy Society. The proceedings of this conference are now available from Pergamon Press, Maxwell House, Fairview Park, Elmsford, New York, 10523. The Canadian Society is holding another meeting next August. It will be on August 22, 23 and 24 at Edmonton, Alberta. In 1977 the Congress of the International Solar Energy Society will be held in New Delhi from November 14-19. Information on membership in the Canadian Society and work being carried out in the fields of Solar and Wind Energy may be obtained from the Solar Energy Society of Canada, Inc. P.O. Box 1353 Winnipeg, Manitoba, R3C 2Z1.

David Phillips of the Atmospheric Environment Service sends this report on the 1977 meeting of Friends of Climatology.

"The 8th Annual Meeting of the Friends of Climatology was held in Toronto on Saturday, March 26, 1977. Close to 50 climatologists from Milwaukee, Toronto, Montreal, Ottawa, and from most universities in southern Ontario were in attendance.

The Friends of Climatology is a non-organization with no constitution, formal membership, officers or dues. A continuum of informal dialogue is maintained through annual assemblies each year to discuss a variety of topics ranging from recent issues in climatology and environmental studies to teaching aids.

The 1977 meeting was organized by Morley Thomas, Gordon McKay, and Dave Phillips of the Atmospheric Environment Service. The theme of the meeting was Climatic Reflections and there was no feature speaker nor formal presentations. Several displays were set up in the main foyer of the AES Headquarters building. These included a permanent display of weather indicators, satellite photographs, current weather maps and information, a variety of data periodicals and statistical publications, and several charts prepared by Terry Allsopp and colleagues, depicting recent weather fluctuations across North America.

Morley Thomas chaired the morning session which was devoted to a discussion of recent developments in climatology and current issues such as climatic change, environmental impact studies, and urban climatology. Hans Lettau travelled the furthest, from Madison, Wisconsin, to describe the potential effect of deforestation on precipitable water content over the tropics and to review progress in climatonomic modelling. After coffee, Howard Ferguson of the AES described the importance of climatology in studies of environmental impact. He called for more interdisciplinary expertise to explain environmental feedbacks and emphasized the need to develop energy strategies for the future. Cynthia Wilson introduced the subject of scale by stating that, although most environmental phenomena and their impacts are local, their solutions are often of a large scale. From the audience, William Baker, a private consultant in recreation, spoke of the need to better relate the physical impacts of development to society and the economy. Two scientists from the Atmospheric Environment Service came forward to discuss climatic change. Gordon McKay, Director of the Meteorological Applications Branch, questioned how we input information on climatic anomalies into the planning system and how we make managers more aware of the strategy and tactical advantage of considering climatic change phenomena. George Boer, of the Research Directorate, described the work of scientists in the AES who are developing diagnostic models of the general circulation and monitoring recent climatic fluctuations. He suggested that recent climatic fluctuations and anomalies of oceanic temperature were not necessarily statistically or physically related. John Lewis of McGill concluded the morning session by criticizing standard climatological networks as being inappropriate to answer urban problems. He remarked that "urban areas are inhomogeneous messes, and in order to measure the environment properly, we must get above the surface." Bruce Findlay of AES responded that it will never be possible to have an adequate urban network. But through careful data selection a useful information base could be made available. He called attention to the danger of the non-climatologist interpreting climatic fields and called for better data presentation and technology transfer.

After lunch, the meeting continued under the direction of Marie Sanderson of Windsor. She guided 17 friends to the platform to speak on a range of topics, from employment opportunities to research funding. Ken King of the Land Resource Sciences Department, University of Guelph, began the afternoon with a review of the 1976 meeting held at Guelph during a symposium on Modelling the Climate - Plant - Soil Complex. The group then heard a report from the orbiting climatologist, John Lewis, on his activities while visiting various universities in Ontario and Quebec in the 1976-77 academic year. The next agenda item, employment of graduates, provoked a lively and spirited discussion. William Baker presented a bleak picture for new graduates in all disciplines. "When times are tough", he said, "planning studies are looked upon as frills and are cut." Gordon McKay and Les Foster from Ryerson Polytechnical Institute tried to present a more optimistic picture. Climatologists trained in geography have made a useful contribution and have demonstrated an ability to interact and communicate well with others outside of their discipline. Revealing comments from two recent graduates, Bob Steward and Ellsworth LeDrew, suggested that over-education and lack of job opportunities were major frustrations for job-seekers. Bob Packer of Western, leaning on many years of experience, suggested that students could not be taught climatological concepts. Questioned on the use of video-tape as a teaching aid, he replied that its main disadvantage is that the listener expects to be entertained. David Phillips of AES described the teaching of applied meteorology to both technical and professional people at AES and offered to compile a work book on climatology lab exercises from contributions sent to him by others in Canada. Professors Miller (Wisconsin), Fraser (Concordia), and Brown (Guelph) talked briefly about the status of climatology in the university. They were followed by three from AES: Morley Thomas, Gordon McKay and Barney Boville and the chairperson, Marie Sanderson. Each described recent developments in climatology at the international level, such as the IGU in Moscow, at WMO and the work of COSAMC and GARP.

The remainder of the afternoon was devoted to the "thorny" issue of research funding for universities. Speakers Rouse (McMaster), McCaughey (Queens) and Jacobs (Windsor) were unanimous in stating that climatology and hydrology have been short-changed when research monies have been allotted in the past. The situation is so desperate that graduate programs in climatology are in jeopardy. It was obvious that more time could be spent discussing the numerous topics affecting the development and usefulness of climatological work in Canada.

The meeting was adjourned at approximately 4:00 p.m. The next "Friends" meeting will be held at either London or Kitchener during the spring of 1978. Interested persons, not already on the mailing list, are invited to contact D.W. Phillips, AES, 4905 Dufferin Street, Downsview, Ontario McH 5T4."

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