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"ATMOSPHERIC REFRACTION AT LOW ALTITUDES"

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"ATMOSPHERIC REFRACTION AT LOW ALTITUDES"

In the development of an emergency navigation method(1) during the recent world war, the basic assumptions on which sunrise and sunset times were (and are) computed for the various Nautical Almanacs were examined. It was found that 34' was allowed for atmospheric refraction at the horizon, regardless of the temperature, the time of year or the latitude. This led naturally to an examination of theories and tables of atmospheric refraction, especially those few which consider low altitudes. A brief survey showed that most of the tables available were based either on the tables of Bessel(2) of 1830 or on other tables almost as old derived from the same observational data which were gathered by James Bradley at an inland observatory (Greenwich) between 1750 and 1762. Furthermore, it has generally been believed that altitudes less than 15° would be unreliable due to the "uncertainty" of atmospheric refraction.

The need for improved tables of refraction corrections at low altitudes in the Arctic was pointed out in 1946(3). In the polar regions, the sun and moon never reach high altitudes, and for intervals of several months, the sun may not reach an altitude as great as 15°. There was in addition the usual scientific reason of "truth for truth's sake".

OBSERVATIONS

In the summer of 1946, a series of measures of the vertical diameter of the sun at low altitudes were made at Smilax, Foster township in western Rhode Island. With these data, a method of obtaining tables of atmospheric refraction was tried out. It is clear that at any time the difference between the horizontal diameter of the sun (which is not affected appreciably by atmospheric refraction) and the vertical diameter (which is) is equal to the differential refraction over the range of altitudes between the lower and upper limbs of the sun. One can start at some altitude at which the refraction is small and well established, and using measures of the vertical diameter of the sun to find the differential refraction, one can integrate downward, obtaining the value of the refraction at a series of points down to the horizon. The first measures were made with a German prismatic reflecting circle reading to 10"; later, modern American endless-tangent-screw sextants with micrometers reading to 0'.1 were used. With instruments of this sort it is possible to observe from the bridge of a moving ship allowing data to be gathered under the same conditions as the tables will later be used.

An expedition to Brazil to observe the total solar eclipse of May 20, 1947, provided an opportunity to try the method out at sea. Nine trained observers made 15,000 observations on the trip from New York to Santos, Brazil and return. A contract with the Office of Naval Research of the U. S. Navy provided the necessary sextants as well as funds for the mathematical reductions of these observations and later ones.

Two tables of refraction corrections for use in the tropics (air temp. 80°F water temp. 80°F, height of eye 0 feet) based on these data, were published in 1950[4] as well as comparisons with the values given by several theories. The most important result is that the deviations from current theories at low altitudes (5° to 2°) are approximately ten times the probable errors of the tabular values derived from the 1947 observations.

The Finn Ronne Antarctic Research Expedition of 1947-8[5] very kindly agreed to make observations for us in the Antarctic. Mr. Harries-Clichy Peterson, physicist of the expedition, was in charge of the observing program; although seriously handicapped by damage to one of their sextants, Peterson assisted by Walter H. Smith, C. O. Fiske and C. H. Hassage measured 1400 vertical diameters of the sun between 33°S latitude and 68°13'S.

To obtain needed observations to the northward, Mr. Andrew Thomson, Controller of the Meteorological Division, Air Services, Department of Transport of Canada, was approached. He suggested that the observers at Eureka and Resolute Weather Stations could help us by gathering data. It was arranged that sextants and second-setting watches were to be sent up to these stations in the summer of 1949; the first observations made at Eureka have been received and preliminary reductions indicate that these observations made in the Arctic region will be of great value in replacing the current inadequate theories of atmospheric refraction.

To supplement the observations from the Eureka and Resolute Stations, a party of four experienced observers, veterans of the Brazil trip, flew to St. Johns, Newfoundland on August 14, 1949, and observed aboard the SS Kyle on the way to Hopedale, Labrador and back. This group consisting of Mr. and Mrs. Donald S. Reed, Miss Mary Quirk and Mrs. Charles H. Smiley made 3,874 measures of vertical diameters of the sun at low altitudes between latitudes of 48°N and 55°N.

Between July 27, and August 18, 1949, through arrangements made by the Office of Naval Research, Dr. Charles H. Smiley travelled from Seattle, Washington, to Pt. Barrow, Alaska, and back to San Diego, California, aboard the USS George Clymer measuring the vertical diameter of the sun whenever the sun was at an altitude less than 10° and the sky was clear. With the generous assistance of a number of young quartermaster strikers as recorders, 1,890 observations of the sun's vertical diameter were made between 43°N and 71°N.

Through the kind cooperation of the Air Weather Service of the Military Air Transport Service, Brig. General Donald N. Yates commanding, a series of observations were made by Smiley and Lt. R. A. Grookett, USAF from planes of the 2078th Air Weather Reconnaissance Squadron, operating out of Fairfield-Suisun (California) Air Force Base; these observations, together with others made earlier by Grookett under similar conditions, proved conclusively that precisely the same kinds of measures which had been made from shipboard could be made from planes, continuing the advantage of observing refraction under conditions similar to those in which the tables would later be used.

Again between August 23rd and September 6, 1949, with the continued generous cooperation of the Air Weather Service, Smiley made a series of measures from regularly scheduled planes of the 375th Air Weather Reconnaissance Squadron, making the Ptarmigan flight across the North Pole and back. While it was shown to be possible to make the observations through an open window, the low temperature and the need for oxygen masks reduced the effectiveness of the observers so much that it was recommended that windows of selected plate glass be installed to permit observations to be made under more favorable conditions. It is hoped that approximately 5,000 measures of the sun's vertical diameter will be made at standard operating elevations on the Ptarmigan flights.

Because of the importance of observations made in the far north, it was planned, in the summer of 1950, to take a party of six veteran observers to Thule, Greenland at 76°30'N for a month; it was hoped that ten or fifteen thousand measures of the sun's vertical diameter at low altitudes could be made from a position near sea level. Through the kindness of the late Charles J. Hubbard, Director of the Arctic section of the U. S. Weather Bureau, the party was to be housed and fed at a reasonable charge by the joint Danish-United States weather station at Thule. Space was requested for the party on MATS planes flying to Thule about August 15th and returning September 15th.

Unfortunately MATS flights to Greenland were cancelled in this interval because of the Korean crisis, and the Greenland observations had to be postponed.

Instead, five of the six observers, Mr. and Mrs. Donald S. Reed, Miss Mary Quirk and Mr. and Mrs. Charles H. Smiley went to the top of Mt. Washington in New Hampshire (6,300 ft. above sea level) and made a series of 8,000 measures of vertical diameter of the sun at low angular altitudes. Originally it was hoped that 5,000 measures might be made in three weeks, but unusually good weather permitted the larger number of observations to be made in ten days.

Besides the postponed Greenland observations, it is planned to have the same group of observers, who have already made some 35,000 measures, make a series of observations from an elevation of approximately 15,000 feet in the tropics, perhaps in Peru. It will, of course, be necessary to compare observations made from a mountain top with others made from a plane to be sure the mountain has not disturbed too greatly the natural distribution of pressures, densities, etc. in its neighborhood.

ACCURACY

To form an estimate of the accuracy of the resulting tabulated refraction corrections, it is necessary to examine the various sources of error in the basic observations and to estimate the order of magnitude of the resulting error in each case. These are the index correction of the sextant and the variation in it, the backlash, the personal equation of the observer and its variation, and the Polaroid filters, which will be taken up in the order named.

The index correction is determined and eliminated by the method of observing, namely by placing the reflected image of the sun alternately above and below the direct image. By averaging an even number of measures of the vertical diameter of the sun made in a given three-minute interval, all counted as positive, the index error is eliminated. The assumption is made that this error does not change appreciably in the three-minute interval; this seems to be verified. Over an observing period of an hour, the error was found to vary appreciably (as much as 0'.8), especially when the sextant was taken from a warm room to a cold out-of-doors observing position.

The effects of backlash are eliminated by the practise of always approaching the final position by turning the micrometer head in the direction of positively increasing readings. This means that negative readings are obtained by bringing the two images of the sun toward tangency with each other while positive readings are found by taking one image of the sun off the other. The spring tension which keeps the endless tangent screw firmly meshed in the threads along the rim of the sextant, tends to minimize the backlash but a number of measures made especially for the purpose indicate that if the convention about a positive final approach is ignored, errors of the order of 0'.1 or 0'.2 are introduced.

The personal equation was found to vary from -1'.0 to 1.0 for different observers while the personal equation of an individual observer may vary as much as 0'.2 or 0'.3 from one observing period to another. The custom was to determine a personal equation for each observer for each observing period by comparing a measured vertical diameter of the sun (at an altitude greater than 10° where possible) with a calculated vertical diameter based on the best available refraction tables for high angular altitudes. Later the personal equation was corrected by insisting that the calculated refraction at the lower limit of observation applied to an observed altitude at that time shall equal the altitude calculated for that time and place.

Contrary to the personal opinions of many navigators, it was found that the index correction of the individual instrument changed appreciably over a period of a few weeks while the personal equation did not vary greatly. The variation of personal equation from observer to observer is still being studied; the curvature of the field of the telescope is believed to be one of several complicating factors.

A number of pairs of Polaroid filters were placed over two openings in the opaque cap of a reflecting telescope of 4" aperture and 25 times magnification and a bright star viewed through the telescope. This test indicated that when used with the usual 3x telescope of the sextant, the errors introduced by the filters would be negligible.

Finally the natural scatter of measures made by a good observer gave a probable error of a single observation of .25', and with about ten observations in a three-minute interval, a probable error of an average vertical diameter of .08'. The aperture of the regular 3x sun-telescope of the sextants was approximately 0.75 inches; this would indicate a theoretical resolving power of 0'.1, if one uses the Dawes value $\frac{4''.5}{a}$, where a is the aperture measured in inches. However this theoretical value is apparently based on the assumption of (a) a perfect telescope, (b) an observer with perfect vision and (c) good seeing conditions. A practical formula would indicate that the aperture of the telescope objective should be twice as large or 1.5 inches to achieve a resolving power of the order of 0'.1. The variation of the values of refraction calculated for a given altitude from measures made in different observing periods gives an indication of the combined effect of the uncertainty of the observations and the variability of refraction from day to day.

THEORY

Because the polar regions are inadequately represented in the observations made to date, it appears unwise to attempt to base a new theory of atmospheric refraction on the data available at this time. However one can draw some conclusions regarding the changes which must be made in current theories of refraction. For example the deviations of these theories from observed refractions in the tropics cannot be explained by a simple change of the temperature coefficient, for values of the refraction based on observations made in the temperate zones do not agree with those made in the tropics at the same air temperature, 80°F. Since Bessel's theory was based exclusively on observations made in the north temperate zone, it is not surprising to find that the observational values obtained in the north temperate zone deviate less from those given by Bessel's theory than do those found in either the polar regions or the tropics.

In this connection, it is interesting to note that in a recent Japanese navigation table[6], refraction corrections to be applied to observed altitudes less than 6° are given which depend on temperature, barometric pressure, and difference between air and water temperatures, while in a Danish navigation table[7] is given a table of corrections to the dip of the horizon depending on the difference of air and water temperatures. Of the two approaches, the former appears to be the better.

In a recent paper[8], Arnold Court has called attention to the need for a method which allows one to take into account, at least in part, the structure of the atmosphere at the time an altitude is measured. He has suggested that in the polar regions, one can use a "refractive temperature" which may differ greatly from the air temperature. Whether this device or some other is eventually adopted for general use, his paper has served a useful purpose in emphasizing the need for just such a simple device which will allow an observer to obtain a reasonably accurate correction for atmospheric refraction at low altitudes.

TABLE I

Altitude	Garfinkel	Smiley	Willis	Bessel
12°	4'.242	4'.291	4'.201	4'.193
11	4 .617	4 .709	4 .574	4'.564
10	5..060	5 .197	5 .013	5 .000
9	5 .587	5 .769	5..539	5 .521
8	6 .229	6 .433	6 .179	6 .155
7	7 .025	7 .287	6 .973	6 .940
6	8 .027	8 .473	7 .981	7 .934
5	9 .337	10 .126	9 .297	9 .230
4	11 .060	12 .353	11 .063	10 .985
3	13 .512	15 .272	13 .550	13 .373
2	17 .065	19 .203	17 .217	16 .942
1	22 .557	24 .182	22 .942	22 .594
0	31 .532	30 .286	32 .540	31 .826

Comparable values of refraction correctionsfor air temperature 80°F., water temperature 80°F. height of eye 0 feet.

In Table I are given comparable values of refraction corrections for the tropics (80°F air temperature, 80°F water temperature, 0 ft. elevation) as given by Bessel's tables[2], Willis's tables[9] with addition from the Pulkovo Tables, Garfinkel's theory[10] and our own integrations. It is to be noted that Bessel's tables go down only to 5° altitude, that corrections for lower altitudes are empirical ones based directly on observations made by Argelander. It should be pointed out that the differences in the empirical portion do not run smoothly, and therefore the values of refraction themselves are open to suspicion.

Willis's table does not provide refractions all the way down to the horizon. As W. D. Lambert has pointed out, even with a special device for increasing the convergence of his series, Willis is able to calculate refraction corrections only down to altitudes of 6° or 7°; refraction corrections for lower altitudes given in the U. S. Coast and Geodetic Survey Special Publication No. 237 are taken from the Pulkovo Tables[11].

Garfinkel's theory provides for the calculation of refraction corrections for altitudes down to zero and even to negative values for cases where the elevation of the observer would permit them to be measured. As published, a considerable amount of work has to be done to obtain usable refraction corrections. According to the observational results presented here, it marks a slight improvement over Bessel's theory, as does Willis's. However the difference between the values given by Bessel's theory and those given by the two recent theories are so small that the use of Bessel's theory can reasonably be continued until a new theory based on modern meteorological data, satisfying recent observational data on refraction and with easily used tables can be developed. To permit the inter-comparison of refractions calculated according to the various theories, the refractions at altitudes of integral degrees from 14° to 5° and for each half degree on down to 0° for temperatures 80°F, 50°F, 30°F, 0°F and -20°F have been computed according to Garfinkel's theory by Miss Barbara Bruce.

One could summarize the work on theories to date roughly thus: most theories have attempted to represent the measured refractions from the zenith to altitudes of 10° say and then again at the horizon. It is to be hoped that in the future, one or more theories will represent also the measured refractions between 10° and the horizon. As pointed out above, more observations, especially in the Arctic and at elevations in all latitudes are needed. From this work will come improved refraction tables and better theories, and as by-products eventually better tables of sunrise, sunset, moonrise and moonset and more practical tables of twilight times.

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